

Position Regulation and Force Transmission in Active Isolation under Axial Thrust and Synchronous Disturbances

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Abstract

The two requirements for an active isolator which is subjected to a mean axial thrust are: to keep the supported mass close to its reference position and to limit the dynamic force applied to the foundation. These requirements are discussed in the context of a 1-DOF Simulink model with a passive spring and damper, a force limit and 1st order actuator dynamics. Under a thrust ramp, periodic loading, speed run-up and temporary overload, passive isolation, PID, LQR and LQI are compared. PID and LQI are able to reduce the displacement to zero (within numerical precision), however in the foundation 20 N are still present. LQR leaves a displacement of about 150 μm . In the run-up interval of 2–20 s, RMS of acceleration is decreased by 9.82 dB using PID, by 14.04 dB using LQR and by 13.89 dB using LQI when compared to the passive one. The frequency-response calculation shows a different comparison: PID provides the largest attenuation near the uncontrolled resonance, but the 25 Hz displacement response is approximately 2.44 times the passive response. Passive-parameter sweeps, order tracking and numerical checks define the circumstances leading to these results. These calculations demonstrate the reasons for considering static load support, position error and dynamic transmission separately. No physical test data are used – all results are numerical.

Keywords

Active Vibration Isolation; Axial Thrust; Displacement Regulation; Order Tracking; Simulink; Transmitted Force.

1. Introduction

An actuator is attached between a supported mass and the foundation, and it will be acting on both. Its force can counteract the movement of the mass and its reaction part can add to the foundation load. This distinction is significant in the case where the isolator supports a continuous force as well as the oscillating disturbance, because a dimensional reduction of the displacement does not determine the amount of force that gets to the foundation. The force balance and feedback description are based on the standard vibration-control theory [1, 2]. The current analysis is about the NVH, structural vibration and transmitted force. It does not include any acoustic-radiation calculation, and the results of its vibration also do not determine the noise reduction.

The passive support is already a compromise in the absence of a controller. The softer spring has a lower natural frequency, but will permit a greater displacement under mean thrust. The resonance response can be reduced by higher viscosity damping but with the risk of higher force transmission at frequencies above the isolation limit. The properties of quasi-zero-stiffness isolators are also a function of load and excitation amplitude [3]. For this case the passive support is linear. It is first

stiffened and damped in various ways to demonstrate which requirements can be satisfied passively and which lead to active positioning.

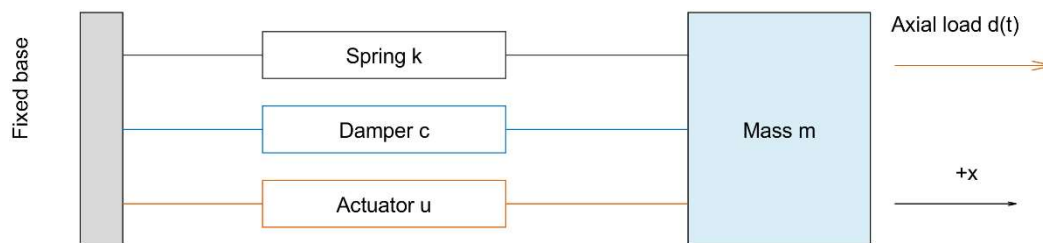
The disturbance-observer-based isolation for optical reference cavities [5] and LQR-based active mounts [4] are examples of previous studies. Their attenuation values are valid for their plants and for their loading. In the present calculation, the mass, stiffness and damping are assumed from those presented in Table 1 in [5] and the fixed-base axial model, thrust inputs and actuator constraints are given separately. The outcome is an equivalent numerical example, not a reconstruction of the apparatus in [5]. So, responses are re-computed for every controller instead of being taken from the published isolation performance.

The second difference is that a speed run-up will result in a changing second excitation frequency (synchronous) while the uncontrolled natural frequency remains fixed in this linear model. Computed order tracking displays the response in relation to the shaft angle and eases the tracking of the synchronous components [6]. It is accurate, depending on the phase information and the interpolation; another accuracy method is the Kalman-based tracking when speed changes quickly [7]. Order and modal descriptions are used for different purposes [8]. An order is related to the number of cycles per revolution and a mode is a quality of the mechanical system. They are used to find the resonance excitation in run-up.

The same mechanical plant and prescribed loads are utilized for comparison between passive isolation, displacement PID, LQR and linear quadratic integral control (LQI). Three questions are asked by the comparison: At what point will the actuator limit prevent position regulation, how does each scheme affect mean thrust, and how does each scheme change the dynamic foundation loading. A Simulink model and calculation files are provided, documenting the numerical procedure. The contribution is this comparison of the operating conditions and assessment measures and not a new control algorithm nor an experimental validation.

2. Axial Model and Transmitted-force Definition

The model shown in Figure 1 has one axial displacement coordinate and the foundation is fixed. The mass is connected in parallel to the foundation by a spring, a viscous damper and a bidirectional force actuator. Positive force is in the positive direction of displacement. The origin of displacement is at the equilibrium prior to the added thrust; gravity load or assembly preload must be absorbed in the reference equilibrium position. Lateral displacement, pitching and foundation motion is not included. The imposed axial forces are entered directly and not assumed to be due to radial rotor imbalance being entered as axial thrust.



Base reaction includes actuator reaction: $F_T = kx + cv - u$

Figure 1. Equivalent axial system and force convention

$$m\ddot{x} + c\dot{x} + kx = d + u \quad (1)$$

In Eq. (1), m is the supported mass, c the viscous damping coefficient and k the equivalent stiffness. The force applied by the actuator on the mass is u , and the external loading is d . Table 1 divides the

mechanical parameters and the assumed actuator parameters and selected numerical parameters. The values $m = 25$ kg, $c = 17.6$ N·s/m and $k = 88,000$ N/m are listed in [5]. Using those values does not reproduce the source apparatus: the boundary conditions and axial reduction adopted here differ. Another drawback of the small value of the damping coefficient is the long settling transient, which is to be taken into account when determining the evaluation period.

$$f_n = \frac{1}{2\pi} \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\sqrt{mk}} \quad (2)$$

From equation (2) the natural frequency is found to be 9.4426 Hz, and the damping ratio is 0.005933, which are the uncontrolled values. The free-vibration envelope has a time constant of $2m/c = 2.841$ s. There is only one mechanical mode in this model. The integral state, PID filter and actuator each introduce additional dynamics into the feedback system; the associated closed-loop poles are not additional structural modes.

$$F_r = kx + c\dot{x} - u = d - m\ddot{x} \quad (3)$$

The reaction of the actuator is included in the total foundation force of equation (3). Without that reaction the 1 of the load paths compared would be missing. When the acceleration is zero (at rest), the equation reduces to equality of foundation force and external thrust. Therefore, after the 20 N foundation load is applied, maintaining a zero displacement under 20 N does not eliminate the 20 N load. When it is in the reference position it takes the static load from the spring and passes it to the actuator.

$$T_{F,p}(r) = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}, \quad r = \frac{f}{f_n} \quad (4)$$

The passive transmissibility in Eq. (4) refers to the dynamic force transmitted to the mass directly when the force is applied to the mass. This is not a base-excitation problem, the foundation remains fixed. In the active cases the same boundary condition is kept. The passive displacement for a constant force F_0 is F_0/k and thus, the stiffness is limited by the allowable displacement. Position and transmitted force must be evaluated separately even with active control as the direct reaction to the foundation is provided by the actuator.

Table 1. Baseline parameters and provenance

Parameter	Value	Basis
m	25 kg	Literature [5]
k	88 000 N/m	Literature [5]
c	17.6 N s/m	Literature [5]
tau	0.003 s	Assumed actuator lag
u_max	40 N	Assumed bidirectional limit
T_aw	0.05 s	Selected anti-windup time
N	200 rad/s	PID derivative filter
dt	0.0005 s	RK4 integration step

3. Controllers and Actuator Implementation

The same external load is applied to each branch and they are all initially at zero conditions. Mechanical parameters and integration step are the same. In passive branch u is equal to zero. In all active branches, command is limited to ± 40 N, then sent through a first-order actuator that has a time constant of 3 msec. The controller gains are calculated or selected for the nominal mechanical plant, then assessed with these actuator dynamics. Once saturation is active, the implementation no longer satisfies the unconstrained linear assumptions of the LQR or LQI design.

$$\tau_a \dot{u} + u = \text{sat}(u_c, \pm u_{\max}) \quad (5)$$

The position reference is zero. The PID law in Eq. (6) uses displacement, low-pass-filtered velocity and an integral-force state. With zero initial conditions, filtering velocity is equivalent to differentiating displacement through the corresponding filtered differentiator. Velocity is available directly from the simulated state. Consequently, these calculations do not test velocity estimation, acceleration-integration drift or measurement-noise rejection. An implementation on hardware would need displacement sensing and a suitable velocity measurement or estimate.

$$\begin{aligned} u_c &= -K_p x - K_d v_f + \eta \\ \dot{v}_f &= N(\dot{x} - v_f) \end{aligned} \quad (6)$$

$$\dot{\eta} = -K_i x + \frac{\text{sat}(u_c) - u_c}{T_{aw}} \quad (7)$$

The PID settings are $K_p = 100,000$ N/m, $K_i = 500,000$ N/(m s) and $K_d = 2,500$ N s/m. These are the gains used in this numerical comparison; no experimental tuning or global optimization is claimed. Equation (7) adds a back-calculation term to the derivative of the integral-force state [9,10]. The term uses the difference between the saturated command and the unlimited command. It therefore responds to command clipping rather than to the difference caused by actuator lag.

$$\begin{aligned} \dot{z} &= Az + B(d + u), \quad z = (x, \dot{x})^T \\ A &= \begin{pmatrix} 0 & 1 \\ -k/m & -c/m \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1/m \end{pmatrix} \end{aligned} \quad (8)$$

$$J = \int_0^\infty (z^T Q z + R u^2) dt, \quad u_c = -Kz \quad (9)$$

The LQR weights are $Q = \text{diag}(10^8, 2,500)$ and $R = 0.01$. They correspond to quadratic normalization scales of 100 μm for displacement, 0.02 m/s for velocity and 10 N for force. Solving the nominal regulator problem gives $K = (45,206.61, 1,566.90)$. The position gain increases the static restoring action, and the velocity gain adds damping. Neither term integrates position error. A constant thrust therefore leaves a nonzero equilibrium displacement, even when the numerical solution has fully settled.

$$\dot{\xi} = x, \quad u_c = -K_x x - K_v \dot{x} - K_\xi \xi \quad (10)$$

For LQI, the displacement integral ξ is added to the state vector. The augmented regulator uses $Q_i = \text{diag}(10^8, 2,500, 10^{10})$ and $R = 0.01$ [11], giving gains of (57,915.91, 1,756.13, 1,000,000). The Simulink implementation keeps the integral contribution as a force, $\eta = -K\xi\xi$, and derivative $-K\xi x$. This is also the state of force that is performed on the integral and the PID back-calculation term is also applied to it. In both controllers, Integral action allows to reject constant thrust position error as long as the thrust available is sufficient and the closed loop is stable. It is not itself a measure of the response to oscillating loads.

This comparison assumes a force limit and actuator lag are the same, but not an equal RMS control effort. The results consequently apply to these gain sets rather than to PID, LQR and LQI in general. There is also a measurement-path difference: PID filters velocity, whereas LQR and LQI use the ideal velocity state. A comparison under a common effort budget and a shared sensor model would require further controller tuning and simulations.

4. Simulink Implementation and Numerical Protocol

Axial force input [N]; output vector [x, v, u, d, uc] in SI units.

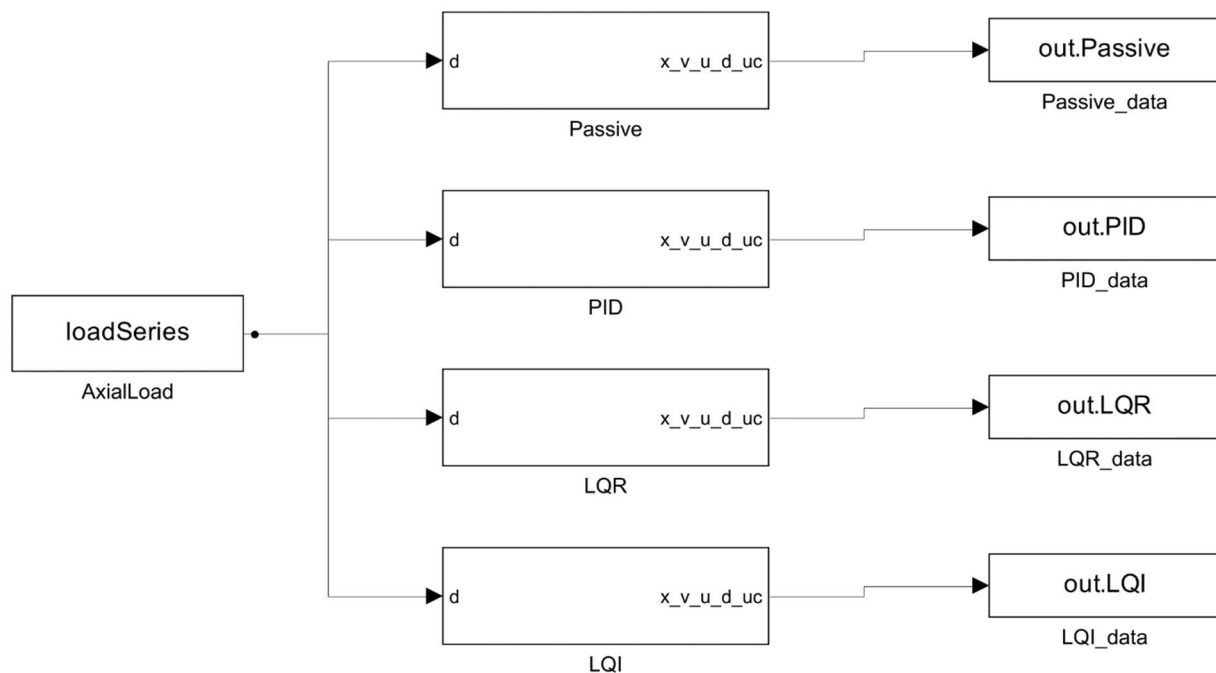


Figure 2. Actual Simulink model with four independently logged branches

The executable model is shown in Figure 2. A From Workspace block reads the external force history and supplies four independent control branches. Within each branch, a State-Space block receives the sum of the applied disturbance and actuator force. The logged signals are displacement, velocity, actuator force, disturbance and unlimited command. Mechanical and control parameters, together with the default run-up input, are stored in the model workspace. The supplied SLX file can therefore run without first populating the MATLAB base workspace.

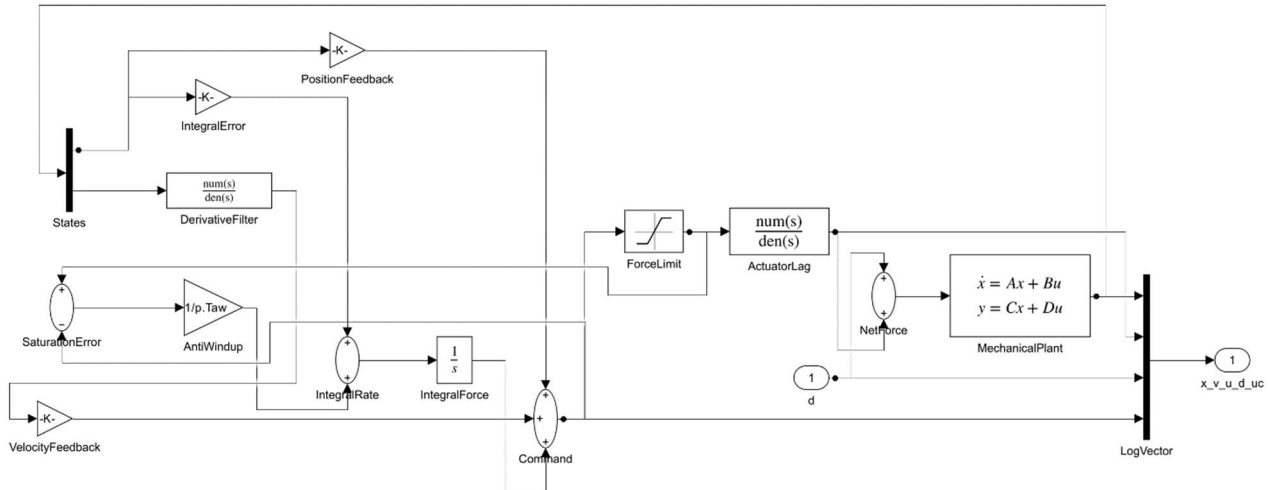


Figure 3. Actual PID subsystem with filtering saturation actuator lag and anti-windup

Figure 3 is exported from the PID subsystem used for the calculations. The command is the combination of PositionFeedback, VelocityFeedback and IntegralForce. It is clipped at ± 40 N, by ForceLimit, and converted to the force applied to the mass by ActuatorLag. AntiWindup and SaturationError provide clipping error to integrator. The LQI branch is identical to the limiting, actuator and back-calculation paths with the gains and without the PID velocity filter.

$$d(t) = 20\rho(t) + A_1 \sin \theta(t) + A_2 \sin(2\theta(t) + \phi) \quad (11)$$

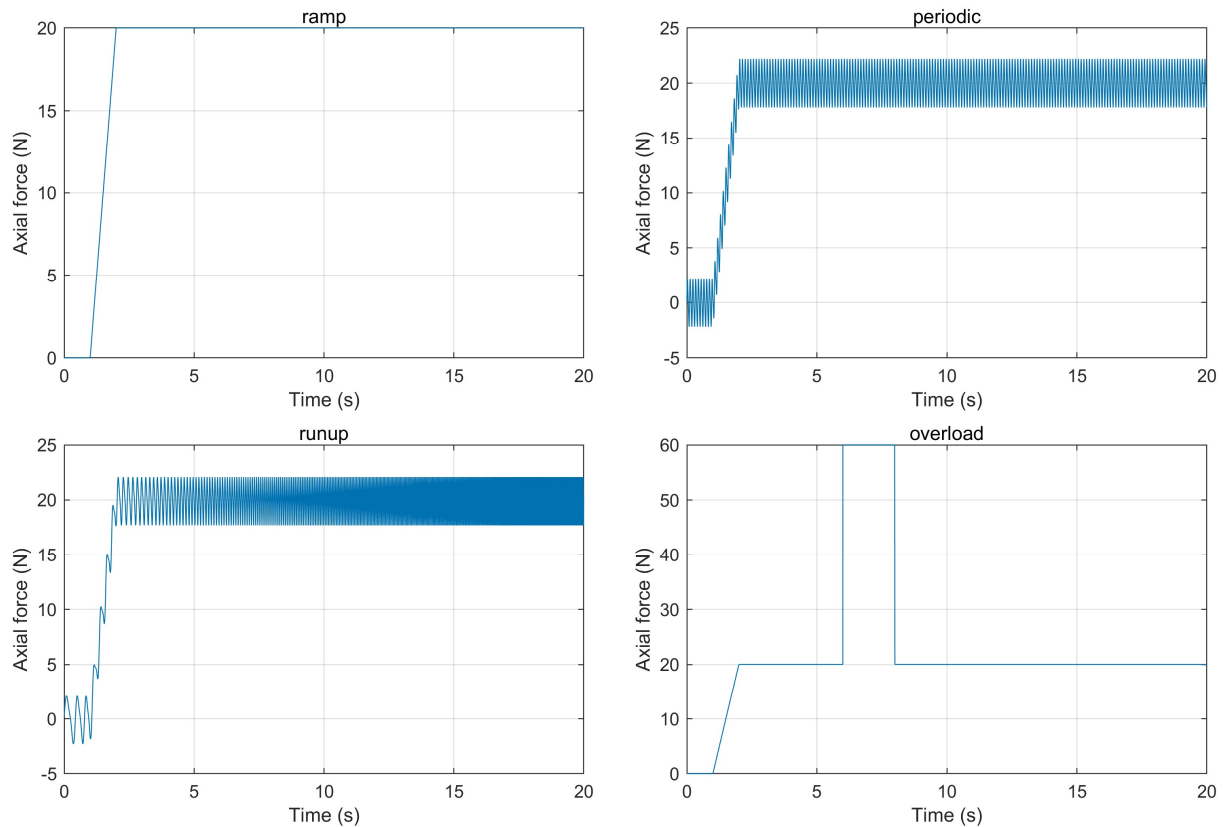


Figure 4. Four prescribed external axial-load histories

Figure 4 shows the four prescribed 20 s input histories. The ramp function $p(t)$ is zero before 1 s, increases linearly from zero to one over 1–2 s, and then remains one. The first case applies $20p(t)$ N alone. The periodic case adds 2 N at 9.443 Hz and 0.5 N at 18.886 Hz. The run-up case, on the other hand, superimposes the first- and second-order forces of 2 N and 0.5 N. The overload case is the same as the 20 N ramp, but with an additional 40 N from 6 to 8 s without the sinusoidal terms. The second harmonic phase is 0 rad in the periodic case, and 0.3 rad in the run-up case. There is no measured load record of any of these inputs.

For run-up, $n(t) = 120 + 84t$ rpm increases speed from 120 to 1,800 rpm over 20 s. Integration of the rotational frequency gives $\theta(t) = 2\pi(2t + 0.7t^2)$. These are the axial synchronous forces generated in this phase. They are maintained at the same amplitude to allow the effects of frequency sweep to be investigated without a concurrent rise in the magnitude of the force applied. The input is thus not an imbalance force proportional to the square of the speed, nor a measured thrust characteristic of the motor or propulsor.

Fixed-step fourth-order Runge–Kutta integration with a step of 0.5 ms (2 kHz output sampling) is used to solve the model. The state of the displacement and velocity is logged. The force balance is used to calculate acceleration and the foundation force is calculated from Eq. (3), both without any numerical differentiation. The step size is checked by repeating the run-up calculation at 0.25ms and comparing the displacements at common run-up times. A separate check applies a 1 N sinusoid at 9.4426, 15 or 25 Hz for 50 s. The last 10 s can be fitted with sine, cosine and constant terms to obtain the amplitude which can be compared with the analytical frequency response.

$$H_{xd} = \frac{G}{1 + GG_a C}, \quad G = \frac{1}{ms^2 + cs + k} \quad (12)$$

$$H_{ad} = s^2 H_{xd}, \quad H_{Td} = 1 - ms^2 H_{xd}$$

In Eq. (12), $G_a(s) = 1/(\tau_a s + 1)$ describes actuator lag. The controller transfer function $C(s)$ is represented as $u_c = -C(s)x$, and contains the gains, integral action and filtering applied by the respective branch. By setting $C = 0$, the passive response is recovered.

The analytical curves are from 0.2 – 100 Hz and include the actuator lag and the PID filter. They do not account for saturation and so represent only the unsaturated response. Nine combinations of stiffness ratios (0.6, 1.0 and 1.4) and damping ratios (0.006, 0.05 and 0.15) are tested in passive design. The outputs calculated include natural frequency, displacement under 20 N and force transmissibility at 25 Hz. The lowest damping ratio in the grid is not exactly the baseline value (0.005933), but has been rounded to 0.006.

$$O = \frac{60f}{n}, \quad n_{cross} = \frac{60f_n}{O} \quad (13)$$

Acceleration is resampled at 128 equally spaced angles per revolution using the known simulated phase. Order spectra are then calculated with an eight-revolution Hann window and 75% overlap. The resulting order-bin spacing is 0.125. FFT magnitudes are normalized by the window sum and expressed as single-sided RMS amplitudes, from which the first- and second-order bins are extracted. DC and Nyquist bins are excluded from amplitude assessment. The speed assigned to each window is that of the window's central angle. It is a computed-order procedure [6, 12] and a tachometer and its measurement errors are not modeled.

The window takes longer to revolve at low speed than at high speed (8 revolutions). It thus sums up the response to various instantaneous frequencies, and the maximum frequency tracked does not have to be at the theoretical velocity for resonance crossing. After a crossing a free-decay component may

also remain near the natural frequency. That fixed frequency component is referred to as an order and will shift in response to a change in rotation speed. Here the model poles obtained from the model are used for the interpretation. It is not an output-only modal identification as this would involve further excitation and identification assumptions [13].

$$\bar{y} = \frac{1}{T} \int_{t_0}^{t_1} y dt, \quad y_{dyn} = \sqrt{\frac{1}{T} \int_{t_0}^{t_1} (y - \bar{y})^2 dt} \quad (14)$$

The mean and dynamic RMS for trapezoidal integration is used in (14): 15-20 s for ramping, periodic and overload loading and 2-20 s for run-up. Peaks use 0–20 s. Saturation occupancy is defined as the percentage of samples having at least $|u_c| \geq 40$ N, from the start of unloading (8 s) until the first sample within $\pm 5 \mu\text{m}$ of the reference is through 20 s. The reference is the arithmetic average of $19 < t \leq 20$ s. Thus, recovery may be terminated at a nonzero position (as with LQR) and evaluated in conjunction with position error.

5. Passive Design and Closed-loop Frequency Response

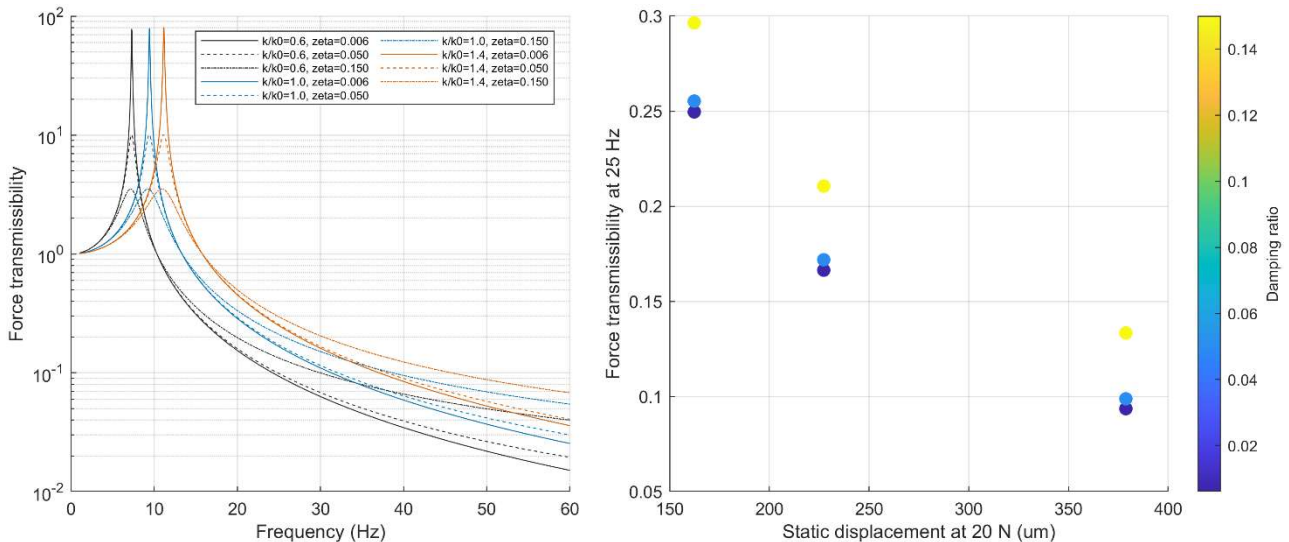


Figure 5. Passive design grid and the static-displacement transmission trade-off

The passive positioning compromise was quantified from the stiffness sweep in Figure 5 and Table 2. The natural frequencies are 7.314 Hz and 11.173 Hz with stiffness 0.6 times and 1.4 times of the original stiffness respectively. Under 20 N, static displacement falls from 378.79 to 162.34 μm . In the case of 0.006 damping ratio, however, the value of force transmissibility at 25 Hz is 0.0937 to 0.2497. The stiffer support gives a smaller fraction of the disturbance at that frequency, but a smaller required axial travel.

Damping gives rise to another trade-off. Increasing its ratio from 0.006 to 0.15 at baseline stiffness decreases the resonance response, but increases 25 Hz transmissibility from 0.1665 to 0.2107. So the damping that is desirable depends on the frequencies of interest when the machine is in operation. When the amount of axial travel is less than the passive static deflection, increase stiffness, or use active positioning. Continuous force capacity is used in active positioning and the passive support and the travel-limit still have to be determined.

Table 2. Passive design results

k/k_0	ζ	f_n Hz	Static x μm	T at 25 Hz
0.6	0.006	7.314	378.79	0.0937
0.6	0.050	7.314	378.79	0.0989
0.6	0.150	7.314	378.79	0.1335
1.0	0.006	9.443	227.27	0.1665
1.0	0.050	9.443	227.27	0.1720
1.0	0.150	9.443	227.27	0.2107
1.4	0.006	11.173	162.34	0.2497
1.4	0.050	11.173	162.34	0.2553
1.4	0.150	11.173	162.34	0.2965

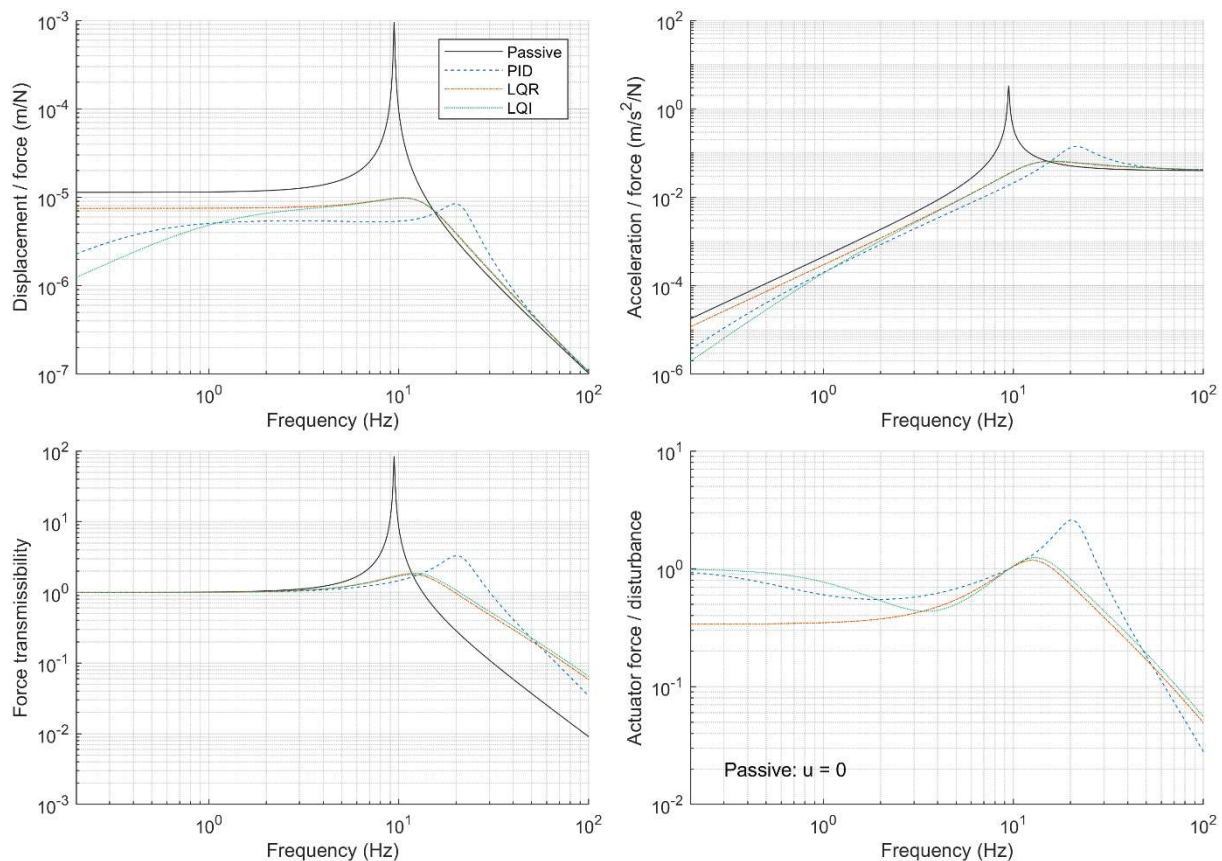


Figure 6. Small-signal closed-loop responses including actuator dynamics

The peak of the passive displacement curve in Fig. 6 is around 9.4426 Hz, which is close to the calculated natural frequency. In this linear model, the amplitude of the response is scaled by altering the amplitude of the force but the mechanical natural frequency remains the same. The actuator and PID filter add dynamics and feedback changes the system poles. The complete closed-loop model must thus be used to examine the peaks in the active curves. Not all of them can be given the primary axial mode.

At 9.4426 Hz, the small-signal displacement reductions relative to passive isolation are about 45.09 dB for PID, 39.85 dB for LQR and 39.96 dB for LQI. The amplitude of the acceleration is ω^2 times the amplitude of the displacement and therefore it has the same attenuation ratio as the displacement at a given common frequency. This comparison changes at 25 Hz: the active displacement amplitudes are about 2.44, 1.21 and 1.24 times the passive amplitude, respectively. Thus, the controller with the deepest reduction near the uncontrolled resonance can still amplify response in another part of the operating band.

The nominal linear closed loops are stable when actuator dynamics are included: all poles have negative real parts. PID has a conjugate pair near $-24.94 \pm 129.32j$ and an integral-related pole near -2.77 . The principal conjugate pairs are approximately $-33.73 \pm 74.33j$ for LQR and $-34.73 \pm 75.07j$ for LQI. The imaginary parts indicate oscillatory frequencies of the poles. They don't mention the exact peaks of each response function, as the peak locations are also influenced by the damping and any transfer-function zeros.

6. Time and Order Responses under Axial Loading

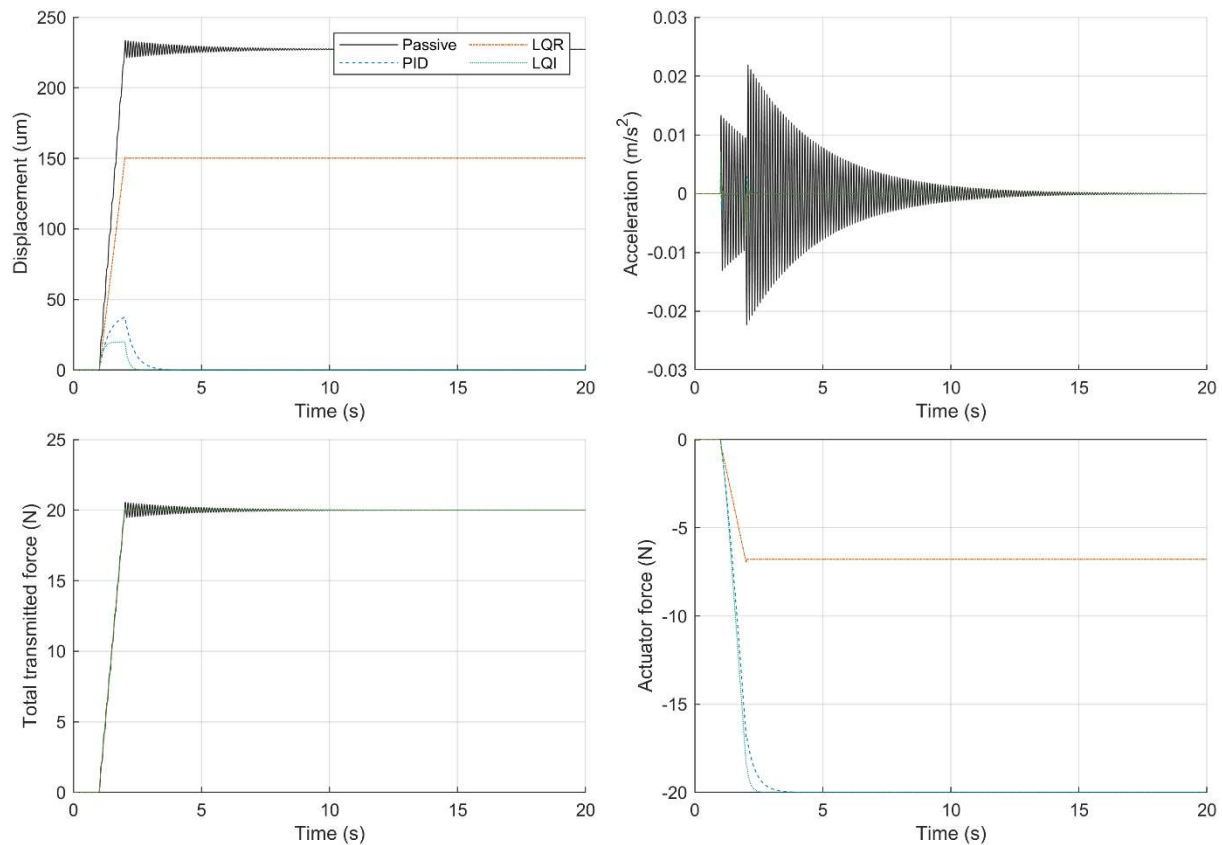


Figure 7. Position regulation and load sharing under the 20 N thrust ramp

The static load sharing is shown in Figure 7 and Table 3 for the thrust-ramp. The final settling point for passive displacement is around 227.27 μm and for LQR is around 150.14 μm. The PID and LQI are close to zero displacement (to the numerical precision) and both provide about -20 N via the actuator. LQR provides -6.79 N and the spring takes up the rest of the thrust. Even though the foundations' positions and the forces that are applied to the actuators are different, all the four foundations support a mean force of 20 N, as specified by Eq. (3).

Table 3. Thrust-ramp results

Scheme	Mean x μm	Peak x μm	Mean u N	Mean F_T N
Passive	227.27	233.63	0.00	20.00
PID	0.00	37.43	-20.00	20.00
LQR	150.14	150.87	-6.79	20.00
LQI	0.00	19.99	-20.00	20.00

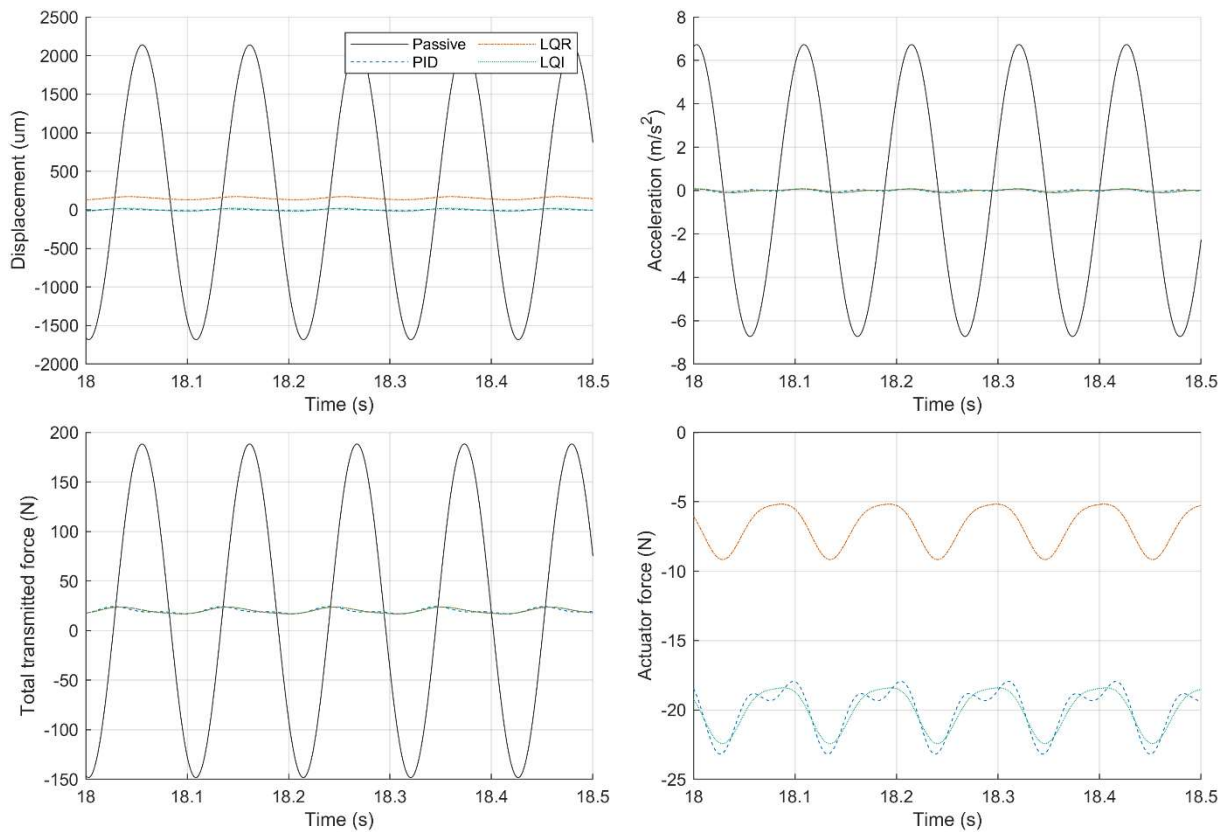


Figure 8. Steady response segment under periodic thrust near resonance

The periodic input places its larger force component close to the passive resonance. In the 15–20 s evaluation window, passive dynamic displacement RMS is 1,348.69 μm and acceleration RMS is 4.748 m/s^2 (Figure 8 and Table 4). The active displacement RMS values are 8.08 μm with PID, 13.86 μm with LQR and 13.69 μm with LQI. Acceleration is also reduced in every active branch. The saved command-saturation occupancy is zero for this case, and peak actuator forces remain below 40 N; these responses therefore compare the unsaturated controller dynamics.

The corresponding dynamic foundation-force RMS falls from 118.69 N in the passive case to 2.29, 2.37 and 2.44 N for PID, LQR and LQI. That is a large reduction relative to the resonant passive response. Nevertheless, all three values exceed the approximately 1.46 N RMS of the combined sinusoidal input. The active schemes reduce resonance amplification without achieving an aggregate dynamic force ratio below unity for this loading. The 20 N mean thrust is a separate load component and is excluded from these dynamic RMS values.

Table 4. Dynamic metrics for periodic and run-up loading

Case	Scheme	x dyn μm	a dyn m/s^2	F_T dyn N	Peak u N
Periodic	Passive	1348.69	4.7478	118.693	0.00
Periodic	PID	8.08	0.0490	2.291	23.18
Periodic	LQR	13.86	0.0532	2.370	9.17
Periodic	LQI	13.69	0.0529	2.435	22.43
Run-up	Passive	108.64	0.3878	9.561	0.00
Run-up	PID	9.34	0.1251	3.045	25.22
Run-up	LQR	9.24	0.0771	1.762	9.31
Run-up	LQI	9.29	0.0783	1.861	22.65

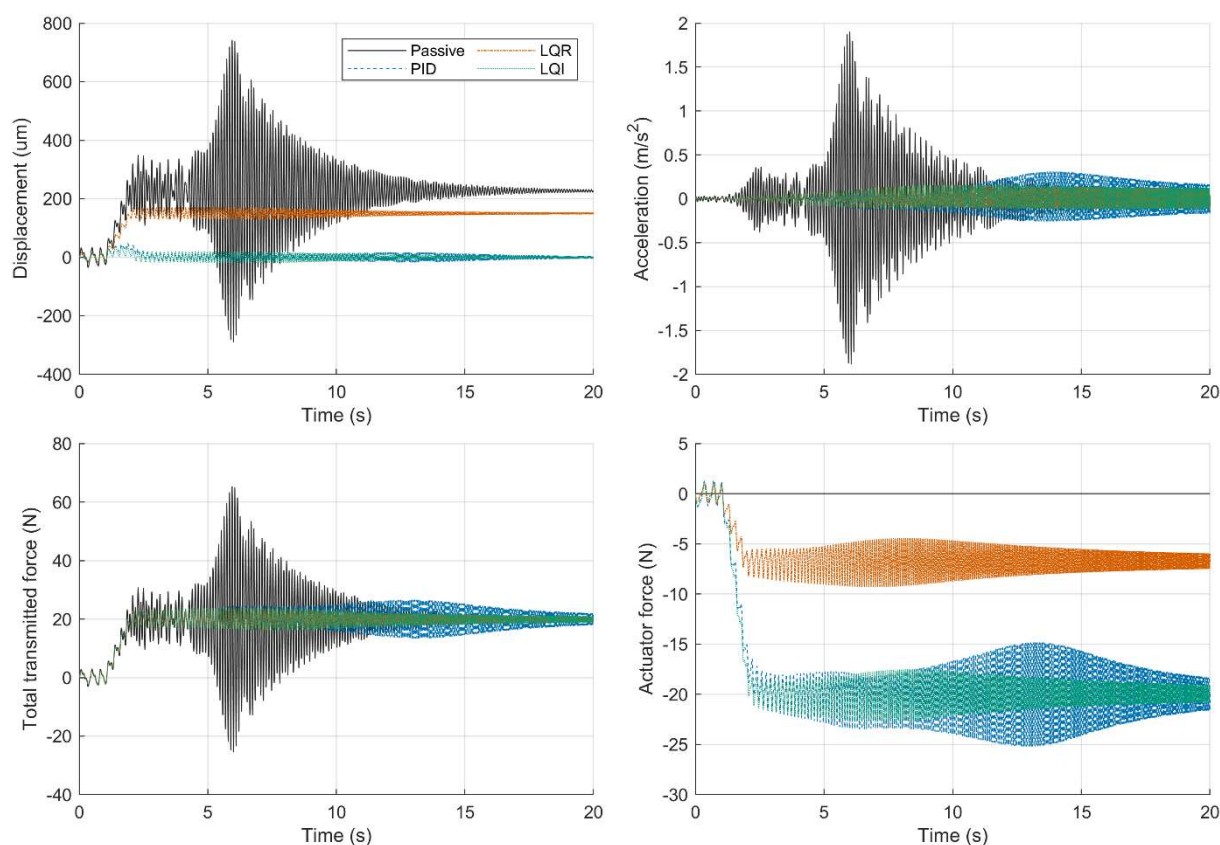


Figure 9. Displacement acceleration transmitted force and effort during run-up

Run-up gives a different ranking from the single-frequency resonance comparison. Over the shared 2–20 s interval, dynamic acceleration RMS is 0.3878 m/s^2 for passive isolation, 0.1251 m/s^2 for PID, 0.0771 m/s^2 for LQR and 0.0783 m/s^2 for LQI (Figure 9). These correspond to reductions of 9.82, 14.04 and 13.89 dB relative to passive isolation. In this run-up, the acceleration by LQR and LQI is lower than that of PID, which has the largest attenuation at the original resonance. The finite-time measure includes the response away from resonance as well as the crossing transient.

The RMS of dynamic foundation force during run-up are, respectively, 9.56 N for passive isolation and 3.04, 1.76 and 1.86 N for PID, LQR and LQI. Of these settings, LQR has the smallest value but its mean position is still displaced by $\sim 150 \mu\text{m}$. LQI eliminates the constant-thrust position error and

generates a somewhat greater dynamic force. Selection therefore depends on whether the position offset is acceptable. When foundation loading is the main design requirement, the controller objective should include transmitted force rather than rely on displacement or acceleration alone.

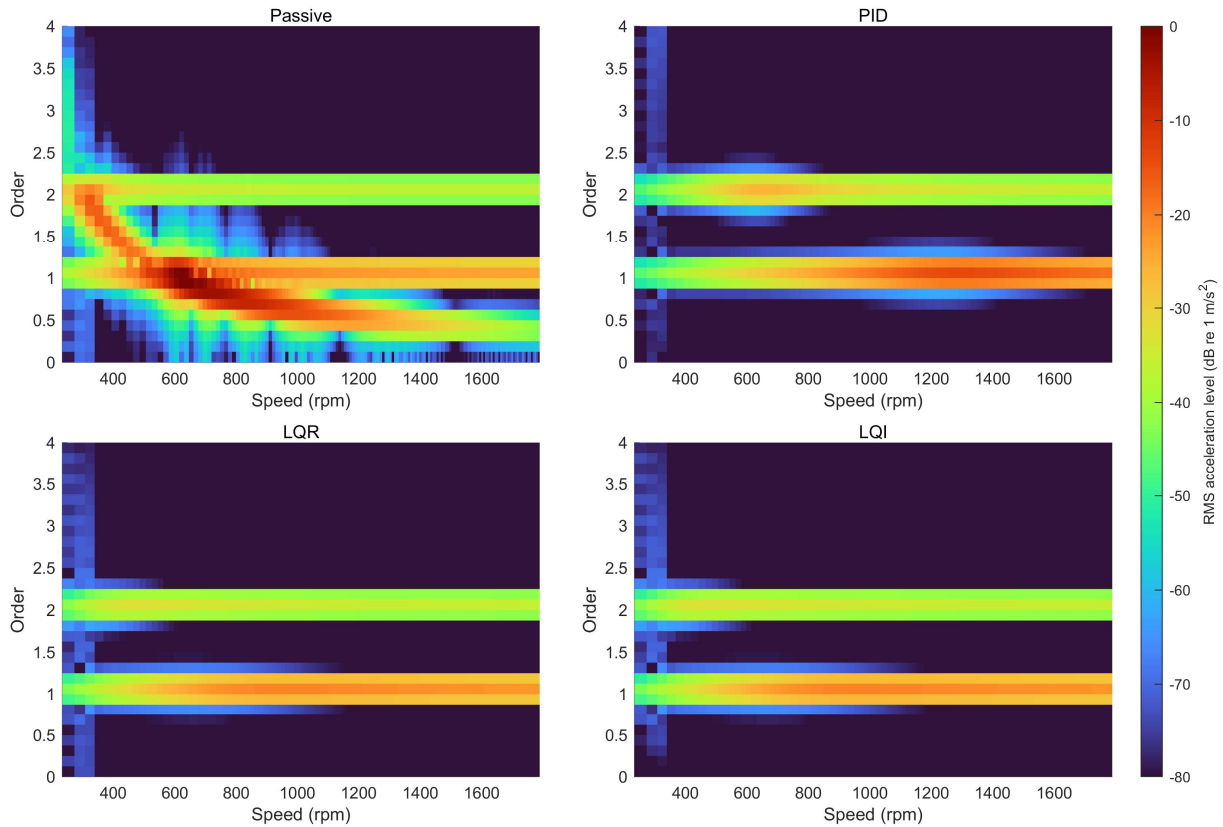


Figure 10. Order–speed maps for the four schemes with a common color scale

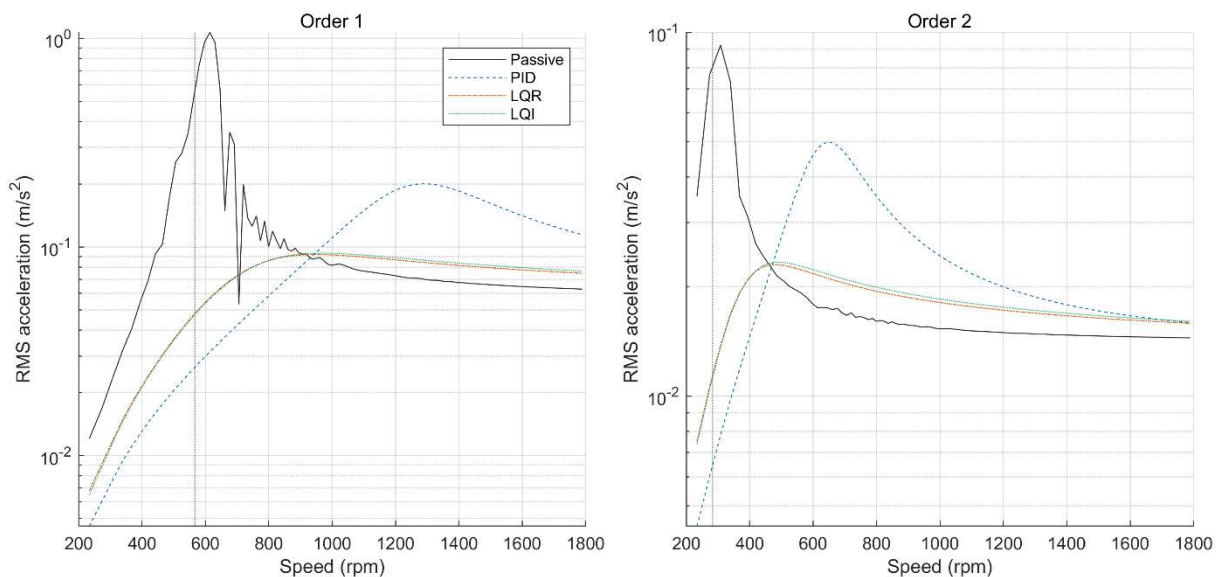


Figure 11. First- and second-order acceleration tracks with uncontrolled resonance crossings

The first and second orders cross the uncontrolled natural frequency at 566.56 and 283.28 rpm. In the order maps and tracks of Figures 10 and 11, the horizontal traces at orders one and two follow the prescribed synchronous forces. Their amplitudes vary because those forces pass through frequency-dependent closed-loop responses. The second-order force has the smaller amplitude, but it sweeps up

to 60 Hz, whereas the first order reaches 30 Hz. It can therefore excite a response band that the first order does not reach.

Table 5 reports the largest tracked RMS amplitude and the speed assigned to its window center. These are maxima from a windowed run-up record, not steady-state FRF peaks or maxima of an averaged order spectrum. No value is extrapolated into the low-speed interval before a full eight-revolution window is available. The exact phase prevents tachometer error in this calculation, but the finite window still combines response from changing operating conditions.

Table 5. Peak tracked order amplitudes

Scheme	Order	Acceleration RMS m/s ²	Window centre rpm
Passive	1	1.06941	614.2
PID	1	0.20088	1291.3
LQR	1	0.09204	949.4
LQI	1	0.09348	970.4
Passive	2	0.09227	308.2
PID	2	0.04995	646.2
LQR	2	0.02298	485.9
LQI	2	0.02334	485.9

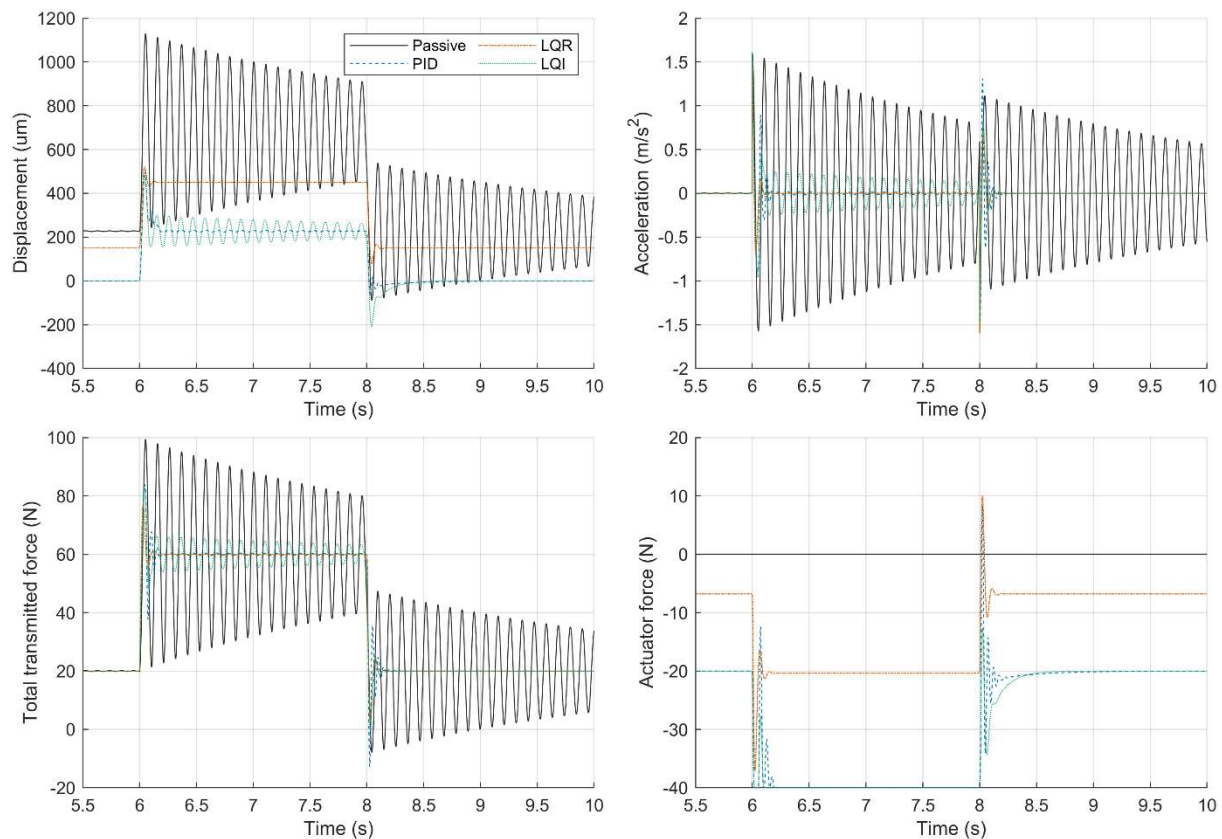


Figure 12. Response to the temporary 60 N overload and subsequent unloading

The overload case is a direct test of the force constraint (Figure 12 and Table 6). The total thrust is 60 N between 6 and 8 s, and the maximum force that the actuator can apply is 40 N. Any load on that spring would be in a static equilibrium, requiring the spring force to be at least 20N, so it is impossible for the spring to be at zero displacement with integral action. The maximum displacements of PID and LQI during the transient are 498.61 μm and 485.50 μm respectively. The failure to hold position follows from the available force, not from the absence of an integral term.

PID and LQI commands reach saturation for 9.46% and 9.78% of the complete 20 s record. Following unloading at 8 s, their recovery times are 0.573 and 0.4695 s under the $\pm 5 \mu\text{m}$ criterion. LQR recovers in 0.105 s, but to its displaced equilibrium near 150 μm ; passive recovery takes about 11.805 s. Back-calculation is enabled in the integral controllers. Since there is no matched calculation with it disabled, these recovery times do not isolate the benefit of anti-windup.

Table 6. Overload and recovery metrics

Scheme	Peak x μm	Peak u N	Saturation %	Recovery s
Passive	1129.46	0.00	0.00	11.805
PID	498.61	40.00	9.46	0.573
LQR	521.17	37.16	0.00	0.105
LQI	485.50	40.00	9.78	0.4695

7. Numerical Checks Parameter Variation and Applicability

Two checks assess the numerical solution (Table 7). Halving the integration step from 0.5 to 0.25 ms changes run-up displacement at common times by less than 0.000191 μm ; the relative RMS difference remains below 3.87×10^{-7} for all four branches. Across the 12 single-frequency comparisons, the largest fitted-amplitude difference from the analytical response is approximately 0.0514%. Differences are the time-stepping, the input-interpolation and finite-window effects. These checks are to ensure that the specified model is consistent between the numerical and analytical calculations, NOT to ensure it is consistent with a physical isolator.

The harmonic checks are made with 40-50s as the passive envelope decays with a time constant of approximately 2.84s. The fit would have more initial free vibration if it was started earlier. The difference between the harmonic response and a residual mean is exhibited by the sine-cosine-constant fit. The definitions of saturation and recovery (Section 4) are used for time integration and are also the definitions for response means. The shorter 15–20 s periodic-case window is still a measure of a finite duration response, which is not considered an exact infinite time steady state.

Table 7. Fixed-gain stability grid and numerical checks

Scheme	Worst pole real part s^{-1}	Relative step RMS error	Max harmonic error %
Passive	-0.1467	2.89e-07	0.0514
PID	-2.5177	3.87e-07	0.0514
LQR	-26.8697	3.74e-09	0.0514
LQI	-6.6216	5.81e-08	0.0514

An experiment is conducted on the stability grid (controller-gains kept fixed while mass and stiffness are varied independently over 0.8, 1.0 and 1.2 times their nominal values). Damping is set to 0.5, 1.0

and 2.0 times its nominal value, representing 27 different combinations. The actuator and for PID, the filter dynamics are included in each pole. All linear systems tested have negative pole real parts, with the smallest negative pole real parts for each scheme shown in Table 7. This is only a pointwise check, not a check for stability between the grid points or for global stability of the saturated system.

Joint identification could be helpful for a practical assembly to obtain the same stiffness and damping that the model has. These parameters can vary with preload, microslip, temperature or vibration amplitude, so the constant k and c are only valid for a local operating condition. Coupled modes or appreciable interface nonlinearity would require a more detailed model and input–output measurements of the control path. Output-only operational modal analysis can inform modal properties, but it does not directly supply a physically scaled force-to-displacement FRF.

The calculation does not include sensor noise, sampling and computation delays, actuator stroke or thermal limits. There is also no flexibility of foundation, and no higher modes is excited. There is no selection of an actuator as the assumed lag is 3 ms and the assumed force limit is 40 N. Specifically, a continuous force of 20 N is not directly an electrical power, it requires the actuator technology and its electrical and mechanical properties. These omissions limit use of the numerical results for hardware selection.

8. Conclusion

- (1) When considering position regulation vs. isolation, the actuator reaction must be taken into account. Although the foundation is still subjected to the thrust of 20 N under these conditions, PID and LQI are able to keep the mass at its reference position. Not static load eliminations, but changes in load sharing is controlled by the controller.
- (2) The passive design grid shows the cost of limiting static travel. As stiffness increases the displacement under mean thrust decreases and the 25 Hz force transmissibility increases for the cases studied. An increase in damping decreases resonance response and increases transmission at that frequency. Passive parameters have to be chosen with respect to travel and operating-frequency requirements, accordingly.
- (3) The comparison of the controller is based on the evaluation condition (3). LQR leaves about 150 μm of constant thrust position error, while LQI eliminates that error and provides a 13.89 dB reduction in the run-up acceleration compared to passive isolation. PID provides the highest small signal attenuation around the uncontrolled resonance with a run-up reduction of 9.82 dB and a 25 Hz response that is greater than the passive response. The results are valid for the gain sets and the force limits indicated only, and not for an arbitrary ranking of the algorithms.
- (4) A 60N overload cannot be balanced by a 40 N actuator at a zero displacement. Limits are exposed by saturation occupancy and recovery to the final position that are not shown by a resonance curve. The numerical solution is complemented by step refinement and harmonic-response checks and is linearly stable for the 27 points in the evaluated parameter grid. If necessary, extension to physical system is needed with coupled-mode modeling, sensor and actuator characterization and experimental verification.

Data and Code

All data are simulations. The MATLAB files that come with this model include the SLX model, build scripts, simulation scripts, MAT files with simulation results, and figure generation code for Simulink and MATLAB R2023b.

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