

Prediction of Bottom-hole Pressure in Deep Wells Considering Temperature-pressure Coupling Effects

Jianfei Wei, Liang Tao*, Xuan Wu

COSL-Expro Testing Services (Tianjin) Company Ltd, Tianjin 300450, China

Abstract

Reliable bottom-hole pressure (BHP) prediction is essential to guarantee operational safety during deep and ultra-deep well drilling. Substantial shifts in drilling-fluid physical properties under high-temperature and high-pressure (HTHP) conditions greatly modify pressure-propagation behaviours inside the wellbore. Conventional prediction approaches commonly treat drilling-fluid properties as fixed constants, which leads to degraded calculation precision when wellbore temperature and pressure experience sharp variations. To tackle this practical limitation, this work constructs a thermo-pressure coupled BHP prediction model that accounts for temperature-sensitive fluid compressibility and rheological parameters. A PVT equation of state is adopted to capture temperature-dependent density variations of drilling fluids, while a modified Arrhenius model describes how rheological characteristics respond to temperature fluctuations. A segment-by-segment iterative algorithm is deployed to solve the coupled system consisting of fluid-property correlations, wellbore temperature-field equations and pressure-gradient equations, yielding the full wellbore pressure distribution. Field data obtained from a 4583-m multi-string cased gas well was employed for model validation. Comparative analyses covering static hydrostatic pressure and circulating-pressure scenarios reveal relative errors ranging from 1.26% to 2.03% against field measurements. Capable of reflecting how thermally induced fluid-property shifts reshape wellbore pressure development, the proposed model fits well with HTHP well pressure-prediction demands. It offers a practical analytical tool for deep-well pressure evaluation and pressure management amid complex drilling scenarios.

Keywords

Deep Well; Bottom Hole Pressure; High-temperature and High-pressure Conditions; Temperature-pressure Coupling; Drilling Fluid Rheology; Iterative Calculation.

1. Introduction

Oil and gas exploration is continuously advancing into deeper geological layers, and deep wells and ultra-deep wells have become important battlefields for the development of oil and gas resources. However, during the drilling process of deep wells, one will directly face high-temperature and high-pressure environments. The available operation window is narrow, and the physical properties of the drilling fluid are greatly disturbed by temperature and pressure. The bottom hole pressure (BHP) is not only affected by static liquid pressure and annular friction, but also the density and rheological properties of the drilling fluid change dynamically with temperature and pressure, which will significantly alter the bottom hole pressure level.

Most traditional calculations of wellbore pressure treat the drilling fluid as an incompressible fluid and assume that the rheological parameters remain constant. This assumption is generally applicable to shallow wells where the temperature and pressure variations are not significant. However, for deep and ultra-deep wells, the combined effects of fluid compression and thermal expansion drive a

significant change in density, and temperature also directly affects the viscosity and flow resistance of the drilling fluid. Ignoring these coupled effects will result in errors in wellbore pressure calculations and increase the risk of failure in controlling drilling pressure [1-3].

A number of studies have conducted analyses of wellbore fluid dynamics under temperature-pressure coupling conditions. Zamora et al. [1] based on experiments of drilling fluid under extreme temperature and pressure conditions, pointed out that the calculation of deep well pressure must take into account the compressibility of the fluid. Demirdal et al. [2] believed that in controlled pressure drilling, the precise characterization of the rheological and volume properties of the drilling fluid is a key prerequisite for pressure control. For gas-liquid two-phase flow under deep water conditions, Song and Guan [4] constructed a temperature-pressure coupling model, confirming that the thermal effect and fluid flow characteristics have a prominent impact on the pressure distribution in the wellbore. Zhang et al. [5] further analyzed the interaction relationship between the temperature-pressure system and the physical properties of the drilling fluid in complex formation environments. Chen et al. [6] developed a fully transient model for predicting coupled wellbore temperature and pressure during underbalanced drilling, accounting for gas-liquid-solid flow, heat transfer, and the thermophysical properties of the circulating fluid. In addition to density evolution, the temperature dependence of the rheological properties of the drilling fluid also alters the annular friction pressure drop. Existing studies mostly use the Arrhenius model to describe the variation pattern of viscosity with temperature. However, drilling fluids are composed of multiple additives in a multiphase suspension system, and the rheological parameters often exhibit complex nonlinear responses under high-temperature conditions. Fikeni et al. [7] investigated the impact of temperature perturbations on BHP prediction, indicating that accurately characterizing rheological properties is the core for improving the reliability of pressure prediction; He et al. [8] conducted high-temperature and high-pressure rheological experiments on water-based drilling fluids and proposed that temperature correction steps cannot be omitted in the hydraulic analysis process. Although there have been many achievements in wellbore temperature-pressure coupling modeling, there are still significant limitations. Some models are oriented towards specific scenarios such as multiphase flow and transient operations, and have insufficient universality, making it difficult to directly apply to deep well pressure prediction; many simplified models still fix the fluid density and rheological parameters, which will underestimate the physical property changes brought about by the thermal-physical coupling in high-temperature and high-pressure (HTHP) conditions; and there is still a lack of an efficient numerical solution framework that takes into account the evolution of physical properties, pressure transmission, and has controllable computational cost.

To address the above issues, this paper develops a deep well bottom pressure prediction model that takes into account the coupling effect of temperature and pressure. The Pressure-Volume-Temperature (PVT) state equation EOS is introduced to describe the nonlinear evolution of the drilling fluid density; based on the Arrhenius model, it is improved to describe the temperature-dependent characteristics of fluid rheological parameters. The coupling of the physical property evolution of the drilling fluid, the temperature distribution in the wellbore, and the pressure transmission mechanism is constructed, and a unit-by-unit iterative solution algorithm is developed. The model is verified using on-site data from a 4583 m well, and the prediction accuracy of the model is evaluated under different working pressure conditions..

2. Mathematical Model

Accurate prediction of bottom-hole pressure requires an appropriate description of drilling fluid properties under high-temperature and high-pressure conditions. In deep wells, the density and rheological parameters of drilling fluids cannot be regarded as constants, but vary dynamically with temperature and pressure. Therefore, the effects of temperature and pressure on fluid properties should be incorporated into the wellbore pressure calculation model.

In this study, the temperature-pressure coupling calculation framework is established by considering the variations of fluid density and rheological properties. The dynamic density model, temperature-dependent rheological model, and pressure gradient equation are introduced to describe the pressure transmission process along the wellbore.

2.1 Dynamic PVT Model for Wellbore Fluid Density

Fluid density is one of the most important parameters determining hydrostatic pressure. In conventional drilling operations, drilling fluids are usually assumed to be incompressible. However, in deep and ultra-deep wells, bottom-hole pressure may exceed 50 MPa, and the compressibility of drilling fluids becomes non-negligible. Meanwhile, elevated temperatures can cause fluid expansion and density reduction.

Fluid-density evolution under high-temperature and high-pressure (HTHP) environments is governed by two counteracting physical mechanisms. Rising pressure compresses intermolecular spacing and raises mass per-unit-volume, which contributes to higher fluid density. By contrast, temperature elevation intensifies molecular thermal motion and expands intermolecular gaps; volume expansion accordingly takes place and drives density downward. Beyond density responses, temperature fluctuations also alter fluid compressibility, giving rise to inherent thermo-pressure coupling interactions.

An empirically-regressed pressure-volume-temperature (PVT) equation of state (EOS) is employed to capture the nonlinear density response of drilling fluid:

$$\rho(P, T) = \rho_0 e^{a(P-P_0) + b(P-P_0)^2 + c(T-T_0) + d(T-T_0)^2 + e(P-P_0)(T-T_0)} \quad (1)$$

where ρ_0 represents the fluid density under reference temperature and pressure conditions (kg/m^3); P_0 and T_0 are the reference pressure and temperature, respectively; P and T denote the pressure (MPa) and temperature ($^{\circ}\text{C}$ or K) at a specific well depth; and a , b , c , d , and e are nonlinear regression coefficients.

Model coefficients are determined by fitting measured density experimental datasets. Regression outputs for water-based drilling fluid are summarised in Table 1.

Table 1. Density model parameters of water-based drilling fluid

Fluid	a	b	c
Water	4.39×10^{-4}	-3.46×10^{-7}	-4.32×10^{-4}

2.2 Temperature-dependent Rheological Model based on Modified Arrhenius Equation

Drilling-fluid rheology dominates frictional pressure loss throughout circulating operations and exerts direct influences on bottom-hole pressure calculation. Reliable quantification of rheological variations induced by temperature is therefore a prerequisite for hydraulic computation in deep drilling scenarios.

Apparent viscosity characterises the overall flow resistance offered by drilling fluid. At fixed shear-rate magnitudes, apparent viscosity generally drops as temperature climbs. Thermal excitation strengthens molecular movement and weakens intermolecular bonding forces, which accounts for the reduced flow resistance. Apart from thermal effects, drilling fluid displays typical shear-thinning features: apparent viscosity declines with increasing shear rate.

The Arrhenius relation is widely adopted to quantify how drilling-fluid rheological parameters shift with temperature:

$$\mu = A_0 \times e^{\frac{E_a}{RT}} \quad (2)$$

where μ is the apparent viscosity (mPa·s); A_0 is the pre-exponential factor; E_a represents the activation energy (J/mol); R is the gas constant (8.314 J/(mol·K)); and T is the absolute temperature (K).

Fitting against lab-measured data yields the corresponding Arrhenius coefficients. Existing literature confirms that the standard Arrhenius formulation works satisfactorily for reproducing viscosity-temperature trends of drilling fluids. Nevertheless, when dealing with complicated drilling-fluid formulations, nonlinear thermal responses may degrade its forecasting performance for yield-stress values.

To enhance prediction fidelity, this work adopts a revised rheological model built upon the Arrhenius framework:

$$\mu = 0.446 \times e^{\frac{13067.18}{T \times 9.314}} \quad (3)$$

Calibration against experimental measurements shows that this revised formulation delivers improved fitting performance, enabling more realistic representation of temperature-dependent rheological shifts within drilling-fluid systems.

During pressure transmission, viscosity determines the magnitude of frictional energy loss. High-viscosity fluids generate stronger shear interactions between adjacent fluid layers, resulting in increased friction resistance and pressure attenuation. Therefore, the pressure loss caused by fluid viscosity is calculated as:

$$P_{fric} = \frac{4\mu h}{dh - dc} \quad (4)$$

where P_{fric} represents the pressure loss caused by viscous resistance (Pa); h is the well depth (m); and d_h and d_c represent the casing inner diameter and drill pipe outer diameter, respectively (m).

2.3 Temperature and Pressure Gradient Model

Assuming that the wellbore wall is adiabatic and the wellbore has reached a stable thermal condition after long-term exposure in the formation, the wellbore temperature is considered equal to the formation temperature.

The geothermal temperature distribution is commonly approximated as a linear relationship with depth:

$$T = 0.025h + T_0 \quad (5)$$

where T represents the formation temperature at a specific depth (°C); h is the vertical depth (m); and T_0 represents the surface temperature.

The pressure distribution in the wellbore is determined by hydrostatic pressure and frictional pressure loss. For a conventional calculation: $P = \rho gh$. However, under HTHP conditions, drilling fluid density varies with both temperature and pressure. Therefore, the pressure gradient should be calculated considering the coupled effects of temperature-dependent density and frictional losses:

$$\frac{dP}{dz} = \rho(P, T)g + \frac{dP_{fric}}{dz} \quad (6)$$

where the first term represents the hydrostatic pressure gradient and the second term represents frictional pressure loss.

For static conditions, the frictional term is equal to zero. For circulating or pressurized conditions, frictional losses contribute to pressure attenuation along the wellbore.

2.4 Element-by-element Iterative Calculation Method

Because temperature, pressure, density, and rheological parameters are mutually dependent, a direct analytical solution is difficult to obtain. Therefore, the wellbore is discretized into finite elements, and an iterative calculation procedure is applied along the well depth.

The calculation starts from the wellhead with known boundary conditions. For each depth element, the local temperature is first determined according to the geothermal model. The corresponding fluid density is then calculated using the dynamic PVT model, followed by updating rheological parameters and frictional pressure loss. The pressure value is iteratively corrected until the difference between two successive calculations satisfies the convergence criterion.

The iterative pressure update is expressed as

$$P_i^{(k+1)} = P_{i-1} + \rho_i g \Delta h + \Delta P_{fric} \quad (7)$$

After convergence, the pressure, temperature, and fluid density distributions along the wellbore are obtained, and the final bottom-hole pressure is determined.

3. Field Case Validation

The applicability of the proposed model was evaluated using field-measured pressure data from a typical deep gas well provided by an oilfield service company. The well has a total measured depth of 4583 m and adopts a multistage casing configuration. To account for the variation in wellbore geometry, different casing inner diameters and drill-pipe outer diameters were assigned to different well sections. The casing inner diameter decreases progressively from 313.6 mm in the upper section to 133.36 mm near the bottom of the well. The main wellbore and operating parameters used in the calculations are summarized in Table 2.

3.1 Wellbore and Operating Parameters

Table 2. Wellbore configuration and operating parameters

Parameter	Section 1	Section 2	Section 3	Section 4	Section 5
Casing inner diameter (mm)	313.6	204	204	133.36	133.36
Drill-pipe outer diameter (mm)	139.7	139.7	101.6	101.6	127
Casing wall thickness (mm)	13.06	20.24	20.24	22.22	22.22
Drill-pipe wall thickness (mm)	19.05	19.05	18.3	18.3	35
Well depth (m)	500	3379	3735	4158	4583

The wellhead temperature was set to 20 °C, and the initial density of the water-based drilling fluid was 1030 kg/m³.

To evaluate the model under different pressure conditions, several field operating cases ranging from relatively low to high applied pressure were selected. Under static conditions, no additional wellhead pressure was applied, and the calculation was mainly used to evaluate the prediction accuracy of

hydrostatic pressure. Under pressurized conditions, different wellhead pressures were applied to represent pressure transmission during field operations and to assess the response of the model to fluid compression and pressure transfer.

The iterative calculation procedure described in Section 2 was used to determine the theoretical pressure at different well depths. The calculated results were then compared with field-measured data obtained from downhole memory pressure gauges or pressure-while-drilling (PWD) measurements.

3.2 Comparison between Predicted and Measured Pressures

The comparison between the calculated and field-measured pressures is presented in Table 3.

Table 3. Comparison between predicted and measured pressure

Parameter	Case 1	Case 2	Case 3	Case 4	Parameter
Depth (m)	3726	3724	3690	3600	Depth (m)
Applied pressure (MPa)	7.18	20.99	28.23	29.14	Applied pressure (MPa)
Measured hydrostatic pressure (MPa)	37.04	37.02	36.691	35.813	Measured hydrostatic pressure (MPa)
Predicted hydrostatic pressure (MPa)	37.63	37.61	37.27	36.36	Predicted hydrostatic pressure (MPa)
Relative error (%)	1.56	1.57	1.55	1.50	Relative error (%)
Rupture-disk pressure (MPa)	44.816	58.605	65.500	65.500	Rupture-disk pressure (MPa)
Measured pressure (MPa)	43.905	57.738	64.639	64.675	Measured pressure (MPa)
Relative error (%)	2.03	1.48	1.31	1.26	Relative error (%)

For hydrostatic pressure prediction, the calculated values show good consistency with field measurements under different depths. The relative errors range from 1.50% to 1.57%, indicating that the temperature–pressure coupled fluid property model can effectively represent the influence of fluid density variation on hydrostatic pressure.

Under pressurized conditions, the predicted pressures also remain close to measured values, with relative errors ranging from 1.26% to 2.03%. The maximum deviation occurs in Case 1, whereas the minimum deviation is observed in Case 4.

The comparison demonstrates that incorporating temperature- and pressure-dependent fluid properties improves the representation of pressure variation in deep wells. Compared with constant-property assumptions, the proposed model can better capture the coupled influence of fluid compression, thermal expansion, and viscosity variation during pressure transmission.

3.3 Discussion of Prediction Deviation

Although the calculated results show good agreement with field measurements, a small systematic positive deviation is observed in all cases. This difference mainly results from several simplifications adopted in the present model.

First, the actual wellbore may experience enlargement or washout during drilling, which increases the annular flow area and changes the local hydraulic resistance. Second, for deviated well sections, differences between measured depth and true vertical depth may influence hydrostatic pressure estimation. In addition, the model assumes that the wellbore temperature reaches thermal equilibrium with the surrounding formation, whereas the actual circulating fluid temperature may deviate from geothermal conditions during field measurements.

These factors may contribute to the observed differences between calculated and measured pressures. Nevertheless, the prediction error remains within approximately 2% under the investigated operating conditions, demonstrating that the proposed model provides reliable pressure estimation for deep-well HTHP drilling applications

4. Conclusion

This paper constructs a temperature-pressure coupling model to predict the bottom-hole pore pressure of deep wells under high-temperature and high-pressure conditions. This model incorporates the density evolution effect related to temperature and pressure and the temperature-sensitive rheological characteristics into the water flow solution process, thereby improving the accuracy of depicting the pressure evolution law within the wellbore. The main research conclusions are as follows:

- (1) In the high-temperature and high-pressure environment, the physical properties of the drilling fluid are significantly affected by the temperature-pressure conditions: The fluid compression effect brought by pressure will cause the density of the drilling fluid to increase, while the thermal expansion effect will cause the density to decrease. If a simplified assumption of constant physical properties is directly adopted, it will cause a significant deviation in the calculation of deep well pressure. This paper introduces the PVT density model and combines the improved Arrhenius rheological constitutive relationship, which can more realistically reproduce the physical property response of the drilling fluid under the combined action of temperature and pressure.
- (2) The iterative algorithm proposed in this paper can synchronously update the wellbore temperature, pressure, drilling fluid density, and rheological parameters along the well depth. A verification was conducted using the field measured data of a 4583 m gas well. The results showed that the static liquid column pressure, circulating pressure, and on-site monitoring data were in good agreement, and the relative error was within the range of 1.26% to 2.03% under the conditions of this set of calculations.
- (3) The model calculation results have a small positive deviation compared to the field measured data. This phenomenon stems from the objective gap between the theoretical idealization assumption and the actual drilling conditions, mainly including wellbore widening, wellbore trajectory influence, and incomplete heat recovery during the circulation heat exchange process. Even with these limitations, this model can still provide a reliable technical means for pressure evaluation and wellbore pressure control in deep high-pressure drilling.

References

- [1] Zamora, M., Roy, S., et al. (2013). Study on the volumetric behavior of base oils, brines, and drilling fluids under extreme temperatures and pressures. *SPE Drilling & Completion*, 28(3), 278–288.
- [2] Demirdal, B., Cunha, J. C., Miska, S., & Takach, N. (2009). Importance of drilling fluids' rheological and volumetric characterization to plan and optimize managed pressure drilling operations. *Journal of Canadian Petroleum Technology*, 48(2), 8–14.
- [3] Aftab, A., Ismail, A. R., Ibupoto, Z. H., et al. (2017). Nanoparticles based drilling muds a solution to drill elevated temperature wells: A review. *Renewable and Sustainable Energy Reviews*, 76, 1301–1313.
- [4] Song, X., & Guan, Z. (2012). Coupled modeling circulating temperature and pressure of gas–liquid two phase flow in deep water wells. *Journal of Petroleum Science and Engineering*, 92–93, 124–131.
- [5] Zhang, R., Li, J., Liu, G., Yang, H., & Jiang, H. (2019). Analysis of coupled wellbore temperature and pressure calculation model and influence factors under multi-pressure system in deep-water drilling. *Energies*, 12(18), 3533.
- [6] Chen, X., He, M., Xu, M., & Zhou, C. (2023). Fully transient coupled prediction model of wellbore temperature and pressure for multi-phase flow during underbalanced drilling. *Geoenergy Science and Engineering*, 223, 211540.

- [7] Fikeni, M., et al. (2022). Effect of temperature variation on the accurate prediction of bottom-hole pressure in well drilling. *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, 46(1), 12575–12595.
- [8] He, M., Shi, H., & Xu, M. (2021). High-temperature and high-pressure rheological behavior of water-based drilling fluids. *Drilling Fluid & Completion Fluid*, 38(3), 271–279.