

Study of A Method for Suppressing Vertical Bending Vibration of High-speed Trains

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Abstract

This paper presents a method to suppress the vertical bending modal vibration of vehicles. The method controls the elastic vibration of the vehicle by placing anti-bending devices longitudinally under the vehicle to suppress vertical bending deformation. In this paper, the dynamic system of a high-speed train with a bending device is established. The influence of the bending device's stiffness and installation position on the vertical bending mode frequency of the vehicle is analyzed, and the variation range of the bending device's parameters is determined. The vibration response of the vehicle system is solved using the numerical integration method. The variation trend of the comfort index is researched by introducing the vehicle's comfort index. The effect of the anti-bending device on the ride comfort of the vehicle and the influence of the anti-bending device on the modal vibration are discussed separately. The results show that the anti-bending device can significantly reduce the vibration amplitude of the vehicle at the vertical bending frequency. When installed in the middle of the vehicle, the anti-bending device can greatly increase the vertical bending modal frequency of the vehicle. The device can significantly affect the ride comfort index of the middle vehicle in the interval of $270 \text{ km}\cdot\text{h}^{-1} \sim 340 \text{ km}\cdot\text{h}^{-1}$. The longitudinal stiffness should be at least $1000 \text{ MN}\cdot\text{m}^{-1}$ to improve the comfort of the middle vehicle under the condition of $300 \text{ km}\cdot\text{h}^{-1}$; and the bending stiffness of $1000 \sim 1800 \text{ MN}\cdot\text{m}^{-1}$ attenuates the vehicle vertical bending vibration when the vehicle speed is lower than $350 \text{ km}\cdot\text{h}^{-1}$. The stiffness of the anti-bending device increases gradually when the speed is higher than $350 \text{ km}\cdot\text{h}^{-1}$. When the stiffness is higher than $1800 \text{ MN}\cdot\text{m}^{-1}$, the device can attenuates the vertical bending vibration effectively within the $250 \sim 350 \text{ km}\cdot\text{h}^{-1}$, and the highest attenuation is 54%.

Keywords

High Speed Train; Vehicle Anti-bending System; Longitudinal Stiffness; Ride Comfort Index; Modal Vibration.

1. Introduction

The lightweight design of high-speed railway vehicles has partially achieved a reduction in the mass of carbody structures, resulting in decreased manufacturing and transportation costs, improved operational speeds, and mitigated track-side noise pollution. However, such light weighting strategies inevitably increase the elasticity of the car body and reduce its natural frequencies. Consequently, the structural elastic vibrations of vehicles within the human sensitivity frequency range become significantly amplified under external excitations. Enhanced elastic vibrations of vehicles not only degrade ride comfort but also elevate the risk of structural fatigue failure. As stipulated in 'Mechanical Vibration and Shock - Evaluation of Human Exposure to Whole-Body Vibration - Part 1', the frequency range in which the human body is sensitive to vertical vibration is 5-15 Hz, and the maximum value is reached at 8 Hz. Studies have demonstrated that the first-order vertical bending

mode of high-speed railway vehicles predominantly governs elastic vibration responses. Notably, the vertical bending modal frequencies of these vehicles typically concentrate within 8-12 Hz, which directly coincides with the human sensitivity band. This frequency overlap critically exacerbates the detrimental effects of vertical bending modal vibrations on ride comfort.

The most straightforward approach to mitigating structural vibration of vehicles involves enhancing vehicle rigidity through measures such as shortening carriage lengths, enlarging cross-sectional areas of vehicles, or increasing corrugated plate thicknesses. However, these methods inevitably compromise transport capacity while increasing manufacturing and operational costs. There is an urgent need for methods that can reduce the elastic vibrations of vehicles without affecting their normal operating conditions.

In order to reduce the elastic vibrations of vehicles, scholars at home and abroad have conducted extensive research on vibration control for high-speed trains. Yang et al. adjusted the bending stiffness and mass distribution in the middle of the train body. When the distance between vertical bending nodes approaches the fixed distance of the vehicle, the level of elastic vibration of the train body is significantly reduced[1]; Chen et al. equated the hanging equipment under the vehicle to a dynamic absorber; and verified through simulation models that the vertical vibration amplitude of the elastic vehicle body can be significantly reduced[2]; Wu et al. proposed a two-stage lateral active suspension technology based on sliding mode control, using a rigid-flexible coupling model. This technology can simultaneously suppress the low-frequency sway and high-frequency elastic vibrations of the vehicle body[3]; Han et al. discovered the main cause of vehicle body vibration and proposed an engineering solution to reduce the transmission of excitation energy[4]; Ning et al. verified the dynamic response characteristics of aluminum alloy car bodies in specific frequency bands, providing a stiffness benchmark for the design of the first-order vertical bending modal frequencies of car bodies[5]; Liu et al. transformed the under-car equipment into power absorbers and proposed a precise method for controlling the elastic vibration of the vehicle body through the design of the suspension parameters of the equipment[6]; Wang et al. focused on the influence of switch impact disturbance on the elastic vibration of the vehicle body, revealing the mechanism of first-order diamond mode resonance induced by switch disturbance, and providing a theoretical basis for line optimization design[7]; Cao et al. revealed the key effects of first-order vertical bending, transverse bending, and diamond-shaped modes on vibration through vehicle modal correction and transfer function design. They proposed improving the stability index by increasing the damping ratio of the vehicle body[8]; Xiao et al. proposed a vibration reduction method based on particle damping, optimized the material, particle size, and filling rate of the damper particles, and ultimately significantly reduced the vibration amplitude of the sensitive area of the vehicle side wall, significantly improving the structural damping efficiency[9]; Dumitriu et al. revealed the influence mechanism of vehicle flexibility on the vertical vibration characteristics. By optimizing the secondary suspension damping ratio, the vibration transmission of flexible vehicles in the vertical bending mode can be effectively suppressed[10]; Sharma et al. simplified the vehicle body as an Euler-Bernoulli beam and studied the effects of equipment mass, suspension stiffness, damping, and installation position on the elastic vibration of the vehicle body[11]; Yamaguchi et al. proposed an improved curved box model, which provides a theoretical tool for optimizing the structural parameters of the carbody. This model reveals that resonance frequencies can be effectively avoided by adjusting the geometric characteristics of the side columns and the stiffness of the roof panel [12]; Dumitriu et al. compared and analyzed the differences between the average comfort index and the Sperling index in the evaluation of vertical vibration of the vehicle body, revealing the key influence of vehicle flexibility and suspension damping parameters on vibration transmission[13]; Leblebici et al. proposed an active suspension control method based on H_{∞} optimization, which achieved collaborative suppression of vehicle floor bending vibration and rigid modes through multi-objective function design[14]; Mazilu et al. designed an anti-bending rod system that uses the reverse torque generated by rigid connectors to counteract the bending deformation of the vehicle body, providing a new idea for lightweight vehicle vibration control[15]; Akiyama et al. proposed a three-dimensional

box-type vehicle analysis model based on continuous systems, which simulates the six degrees of freedom motion of each panel of the vehicle body by introducing anisotropic elastic bodies. This model achieves high-precision prediction of the target modal frequency on the Shinkansen test vehicle [16]; Melero et al. conducted experimental research on constrained layer damping structures and adopted a high-inertia honeycomb constrained layer fully-wrapped scheme in the lightweight design of the vehicle body to maximize absolute damping. This provides an experimental basis for balancing weight reduction and vibration suppression [17]; Palomares et al. first used frequency-weighted acceleration as an adaptive optimization index for aerodynamic suspension, combined with the first two bending modes of the vehicle body to construct a model and solve the constrained optimal control problem, significantly improving vehicle comfort [18].

This paper proposes a methodology for suppressing vertical bending vibrations in high-speed railway vehicles. By installing anti-bending devices on the chassis of the vehicle, the vertical bending deformation of the vehicle is suppressed, the bending stiffness and first-order bending natural frequency of the vehicle are increased, the vertical bending mode vibration of the vehicle is reduced, and the ride comfort of the vehicle is improved. The findings provide theoretical and practical guidance for elastic vibration control in railway vehicle engineering.

2. Vehicle System Modeling with an Anti-bending Device

The schematic of the railway vehicle system integrated with an anti-bending device is illustrated in Figure. 1. The anti-bending device is longitudinally installed on the carbody underframe, generating counteracting forces or moments to suppress vertical bending modal vibrations during operation.

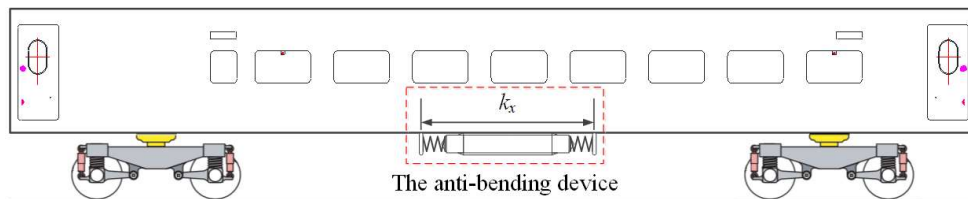


Figure 1. Schematic of vehicle structure with anti-bending device

The dynamic model of the high-speed railway vehicle is depicted in Figure. 2. The vehicle system consists of one body, two frames, and four wheelsets. The wheel-rail contact is assumed to maintain full adhesion. Key parameters include: v is the vehicle speed; L is the length of the vehicle; m_c is the mass of the vehicle body; J_c is the moment of inertia of the vehicle body pitching; m_b stands for the vehicle's weight; J_b is the moment of inertia of the framework pitching; l_1 and l_2 are the positions of the secondary suspensions at both ends, respectively; l_{e1} and l_{e2} are the front and rear suspension positions of the anti-bending device; a_c is half of the vehicle's fixed distance; h_1 is the vertical distance between the center of gravity of the vehicle and the longitudinal suspension of the secondary system; h_2 is the vertical distance between the longitudinal suspension of the secondary system and the center of gravity of the frame; h_3 is the vertical distance between the center of gravity of the framework and the longitudinal suspension system; a_b is half of the framework spacing; k_{bz} and k_{cz} represent the vertical stiffness of the primary and secondary systems, respectively; c_{bz} and c_{cz} are the first and second series vertical damping, respectively; k_{bx} and k_{cx} represent the longitudinal stiffness of the first and second series, respectively; c_{bx} and c_{cx} are the first and second series longitudinal damping, respectively; $k_{c\theta}$ and $c_{c\theta}$ represent the pitch stiffness and damping of the secondary suspension system; m_w is the mass of the wheelset; $w(x,t)$ is the vertical vibration displacement of the vehicle body, including rigid body displacement and elastic displacement; x is the longitudinal position coordinate of the vehicle body; t is the running time; z_c is the bounce displacement of the vehicle body; θ_c is the pitch displacement of the vehicle body; z_{b1} and z_{b2} represent the bounce displacement of the front and rear end structures, respectively; θ_{b1} and θ_{b2} represent the pitch displacement of the front and

rear end frames, respectively; x_{b1} and x_{b2} represent the longitudinal displacement of the front and rear end frames, respectively; z_{w1} , z_{w2} , z_{w3} , z_{w4} are the vertical displacements of wheelset 1, wheelset 2, wheelset 3, and wheelset 4, respectively; x_{w1} , x_{w2} , x_{w3} , and x_{w4} have longitudinal displacements of wheelset 1, wheelset 2, wheelset 3, and wheelset 4, respectively.

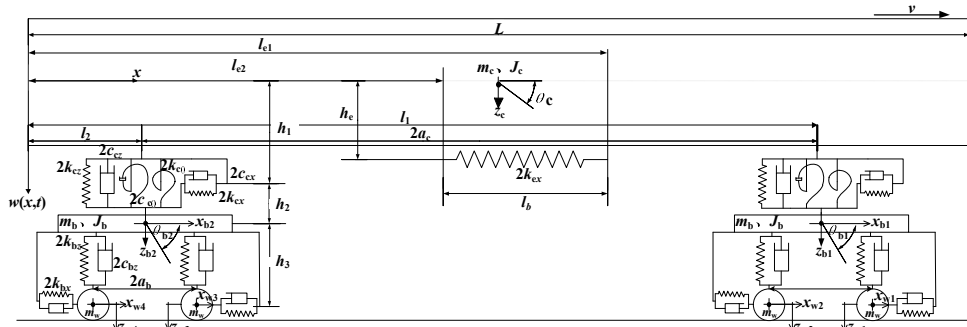


Figure 2. Dynamic model of the vehicle system

The vertical vibration displacement of the carbody is formulated as the superposition of three modal components:

$$w(x,t) = z_c + (x - 2/L)\theta_c + X_1 T_1 \quad (1)$$

In the formula: T_1 is the vertical bending mode coordinate of the vehicle body; X_1 is the vertical bending matrix function of the vehicle body. The carbody equation of motion:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \mu I \frac{\partial^5 w(x,t)}{\partial t \partial x^4} + \rho \frac{\partial^2 w(x,t)}{\partial t^2} = \sum_{i=1}^2 \left[F_{czi} \delta(x - l_i) + (M_{ci} - h_i F_{cxi}) \frac{d\delta(x - l_i)}{dx} + h_c F_{exi} \frac{d\delta(x - l_{ci})}{dx} \right] \quad (2)$$

In the formula: E is the elastic modulus of the carbody; I is the moment of inertia of the carbody cross-section; μ is the damping coefficient of the vehicle structure; ρ is the mass per unit length of the carbody; $\delta(x)$ is the Dirichlet function; F_{cz1} and F_{cz2} are the vertical forces acting on the front and rear suspension systems, respectively; M_{c1} and M_{c2} respectively represent the torque exerted by the front and rear suspension systems on the carbody; F_{cx1} and F_{cx2} are the longitudinal forces acting on the front and rear suspension systems, respectively; F_{ex1} and F_{ex2} are the longitudinal forces acting on the bending device, respectively.

Bounce motion (front/rear bogies):

$$m_b \ddot{z}_{b1} = \sum_{k=1}^2 F_{bz k} - F_{cz1} \quad (3)$$

$$m_b \ddot{z}_{b2} = \sum_{k=3}^4 F_{bz k} - F_{cz2} \quad (4)$$

Pitch motion (front/rear bogies):

$$J_b \ddot{\theta}_{b1} = \sum_{k=1}^2 \left[a_b (-1)^{k+1} F_{bzk} - h_3 F_{bxk} \right] - M_{c1} - h_2 F_{cx1} \quad (5)$$

$$J_b \ddot{\theta}_{b2} = \sum_{k=3}^4 \left[a_b (-1)^{k+1} F_{bzk} - h_{b1} F_{bxk} \right] - M_{c2} - h_{b2} F_{cx2} \quad (6)$$

In the formula, F_{bzk} represents the vertical force exerted by the suspension system; F_{bxk} is the longitudinal force acting on the suspension system.

Longitudinal motion (front/rear bogies):

$$m_b \ddot{x}_{b1} = \sum_{k=1}^2 F_{bxk} - F_{cx1} \quad (7)$$

$$m_b \ddot{x}_{b2} = \sum_{k=3}^4 F_{bxk} - F_{cx2} \quad (8)$$

Longitudinal motion of wheelsets:

$$m_w \ddot{x}_{wk} = -F_{bxk} \quad (9)$$

Bounce motion of wheelsets:

$$m_w \ddot{z}_{wk} = -F_{bzk} \quad (10)$$

3. Mapping the Relationship between Anti-bending Device Parameters and Vertical Bending Modal Characteristics

Based on the vehicle system model in Figure. 2, the vertical bending modal frequency of the vehicle equipped with the anti-bending device is derived as:

$$f_c = \frac{1}{2\pi} \sqrt{\frac{K_c + 8k_{ex} X(l_{e1})^2 h_e^2}{M_c}} \quad (11)$$

In the formula, M_c and K_c respectively represent the vertical bending modal mass and stiffness of the vehicle; k_{ex} is the longitudinal stiffness of the anti-bending device; $X(l_{e1})$ is the derivative of the vehicle formation function at l_{e1} ; h_e is the vertical distance between the vehicle's center of gravity and the anti-bending device. Figure 3 illustrates the relationship between the stiffness of the anti-bending device and the vertical bending modal frequency. Key observations include: Without an anti-bending device, the first-order vertical bending frequency of the vehicle coincides with the original first-order vertical bending modal frequency of the vehicle. Increasing longitudinal stiffness of the anti-bending device elevates the first-order vertical bending frequency of the vehicle. When the longitudinal stiffness is set to 3000 MN/m and the vertical bending mode frequency reaches 21 Hz, it can be seen

that the longitudinal stiffness of the anti-bending device has a significant effect on increasing the vehicle's vertical bending frequency, with a maximum increase of 70%.

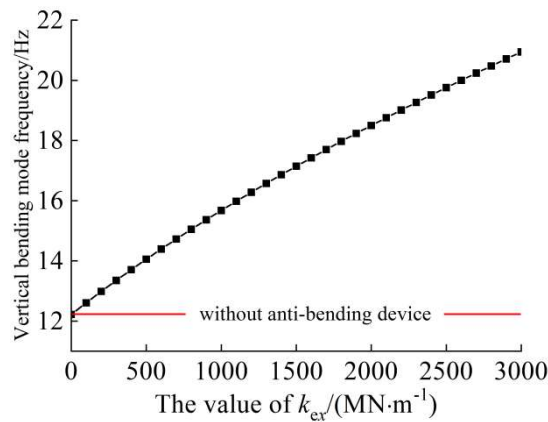


Figure 3. Vertical bending frequency versus anti-bending device stiffness

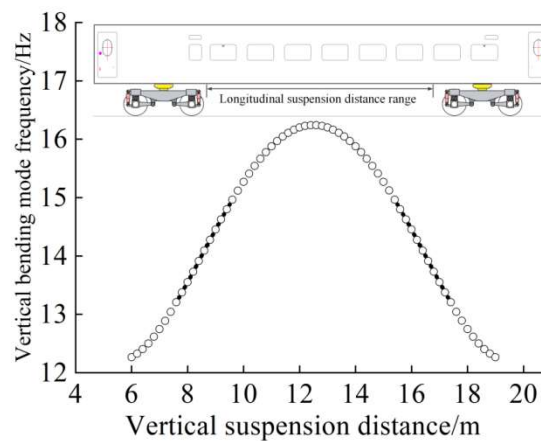


Figure 4. Vertical bending frequency versus anti-bending device installation position

The influence of the anti-bending device's longitudinal installation position on the first-order vertical bending frequency of vehicles is shown in Figure. 4. Critical findings are: The frequency variation exhibits symmetry about the carbody's longitudinal midpoint. Mounting the device closer to the carbody ends reduces its frequency-elevation effectiveness. As the installation position approaches the middle of the vehicle, the vertical bending modal frequency of the vehicle gradually increases and reaches its maximum value (16.2 Hz) when located in the middle of the vehicle. It can be seen that the anti-bending device has the most significant effect on increasing the vertical bending mode frequency of the vehicle when it is located in the middle of the vehicle. However, the actual space structure under the vehicle is complex, and it is not realistic to increase the vertical bending frequency by changing the device's position. The following will take the installation of anti-bending devices in the middle of the vehicle as an example for research.

4. The Influence of Anti-bending Device Parameters on Vehicle Vibration Characteristics

After installing the anti-bending device, it is also necessary to consider whether the device itself resonates with the vertical bending mode of the vehicle. The research results in reference [19] indicate that when the natural frequency of the anti-bending device is at least twice the vertical bending mode frequency of the vehicle, resonance between the two can be avoided. Therefore, the formula for calculating the natural frequency of the bending device is:

$$f_c = \frac{4.73^2}{2\pi l_b} \sqrt{\frac{k_{ex}}{4\pi\rho l_b}} \quad (12)$$

In the formula: l_b is the length of the anti-bending device; ρ is the device density. By combining Equations (11) and (12), we can obtain.

$$k_{ex} \geq \frac{16K_c l_b^3 \pi \rho}{4.73^4 M_c - 128 l_b^3 \pi \rho X(l_{el})^2 h_c^2} \quad (13)$$

According to Equation (13), the flexural stiffness is related to the material density, length, and suspension position. In practice, the material density and suspension position of the device remain unchanged, and the stiffness is mainly determined by the length. Assuming the length range of the device is 1-5 m, the stiffness variation range of the device can be calculated to be approximately 20-2800 MN·m⁻¹. This range is also the basis for the following calculations.

In order to reveal the influence of bending device parameters on the vibration characteristics of vehicles, a new prediction correction integration method is introduced in this paper to solve the system shown in Figure 2[20]. During the integration process, the unevenness spectrum of the Beijing-Tianjin intercity rail is used as the wheel-rail excitation. Figure 5 shows a comparison of the vertical bending modal vibration response of vehicles with and without anti-bending devices at a speed of 200 km/h. Perform vibration spectrum analysis based on the calculation results shown in Figure 5, as shown in Figure 6.

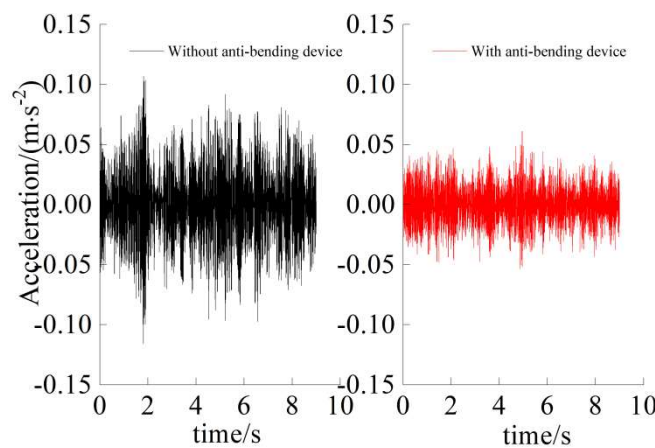


Figure 5. Comparison of time-domain response of vertical bending mode vibration with and without anti-bending device, a) without anti-bending device; b) with anti-bending device.

As shown in Figure 5, the vertical bending mode vibration of the vehicle was effectively suppressed after the installation of the anti-bending device. The effective values of the vertical bending mode vibration of the vehicle with and without the anti-bending device were 0.0159 m·s⁻² and 0.0307 m·s⁻², respectively. The maximum values of vertical bending vibration in the middle of the vehicle are 0.0611 m·s⁻² and 0.1068 m·s⁻², respectively. It can be seen that the anti-bending device has a good suppression effect on the vertical bending vibration of the vehicle under this working condition. Figure 6 shows the vibration acceleration spectrum of Figure 5. It can be seen that after installing the anti-bending device, the vertical bending mode frequency of the vehicle increased from 10.8 Hz to 13.9 Hz, and the vibration amplitude at the vertical bending frequency decreased by 52%.

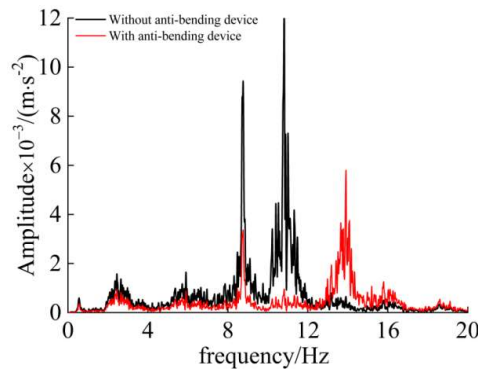


Figure 6. Comparison of vertical bending modal vibration spectra with and without anti-bending devices

The comfort index is a comprehensive indicator for evaluating the riding performance of a vehicle. It is a weighted comprehensive evaluation of the acceleration in the corresponding direction of the vehicle body measurement points. The comfort index, as an indicator of vehicle ride performance, has been widely used internationally. In this article, the comfort index is used to evaluate the comfort of vehicles. According to the regulations on vibration measurement points in "UIC 513: Guidelines for evaluating passenger comfort in relation to vibration in railway vehicles", reference points are selected as shown in Figure 7, where #1, #2, and #3 are the reference points for the comfort indicators of both the end and middle of the vehicle, respectively.

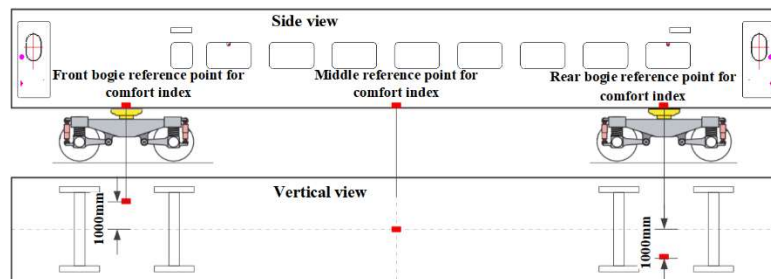


Figure 7. Schematic diagram of reference points for vehicle comfort indicators

Figures 8 to 11 calculate the impact trend of the longitudinal stiffness of the bending device and changes in vehicle operating speed on the vehicle comfort index. Figures 8 and 9 show the trend of changes in the comfort index at the end of the vehicle. It can be seen that the comfort index gradually increases with the increase of vehicle speed, and the longitudinal stiffness of the anti-bending device has no significant effect on the comfort index at the end of the vehicle. Figure 10 shows the trend of changes in the comfort index of the vehicle's middle section. It can be seen that, in addition to the vehicle's operating speed, the longitudinal stiffness of the anti-bending device also affects the comfort of the middle section. The higher the vehicle speed, the more significant the impact.

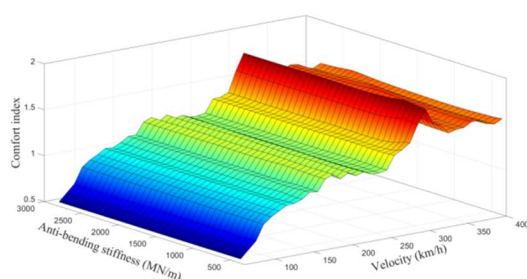


Figure 8. Comfort index above the front bogie

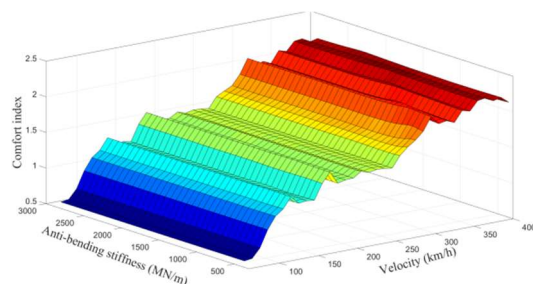


Figure 9. Comfort index above the rear bogie

In order to gain a more intuitive understanding of the degree of impact, Figure 11 lists the operating conditions that have the most significant impact on comfort indicators. The negative sign on the y-axis in Figure 11 indicates a decrease in the comfort index, and the larger the absolute value of the y-axis, the more significant the decrease in the comfort index. Based on the calculation results in Figures 10 and 11, it can be concluded that: The stiffness of the anti-bending device has the greatest impact on the comfort index of vehicles in the middle of the speed range of $270 \text{ km} \cdot \text{h}^{-1}$ to $340 \text{ km} \cdot \text{h}^{-1}$. The greater the longitudinal stiffness of the anti-bending device, the more the comfort index in the middle decreases (up to 6.7%). When the vehicle speed is around $300 \text{ km} \cdot \text{h}^{-1}$ and the stiffness is at least $1000 \text{ MN} \cdot \text{m}^{-1}$, the influence of the anti-bending device on the comfort of the vehicle center gradually becomes prominent.

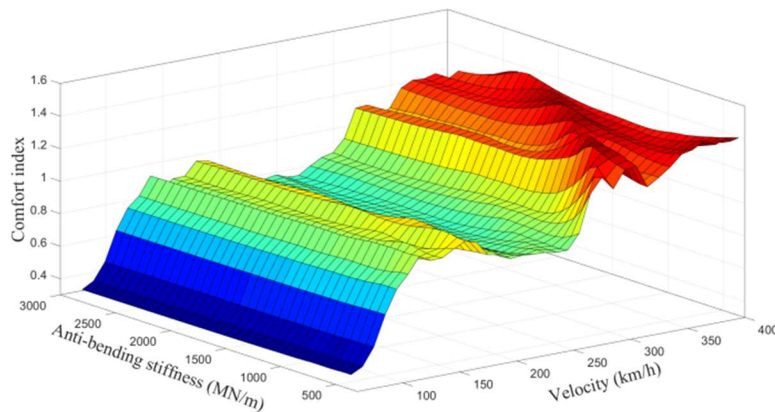


Figure 10. Comfort index in the middle of the vehicle

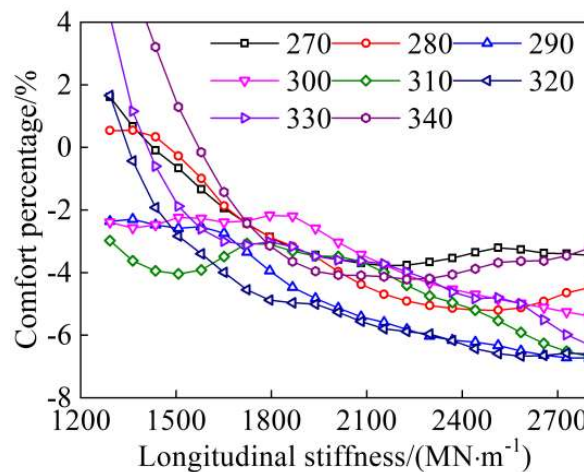


Figure 11. The changes in comfort level under the main operating conditions

The vibration response of the vehicle in the text is the superposition of body bounce, pitching, and vertical bending modal vibrations. In order to clarify the relationship between the longitudinal stiffness of the anti-bending device and the different modal vibrations of the vehicle body, Figures 12-14 calculate the trend of the third-order modal vibration of the vehicle body as a function of vehicle speed and the stiffness of the anti-bending device.

From Figures 12 and 13, it can be seen that the effective values of bounce mode vibration and pitching mode vibration of the vehicle body gradually increase with the increase in vehicle speed. However, they are not significantly affected by changes in the stiffness of the bending device. The longitudinal stiffness of the bending device has little effect on the rigid mode vibration of the vehicle body.

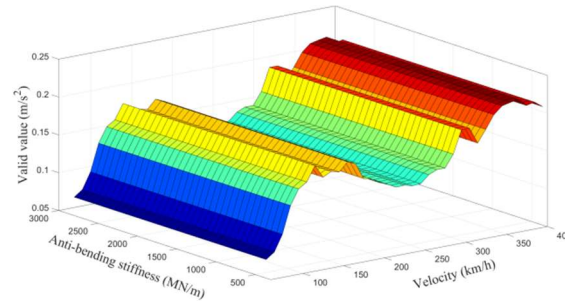


Figure12. Effective value of bounce mode vibration

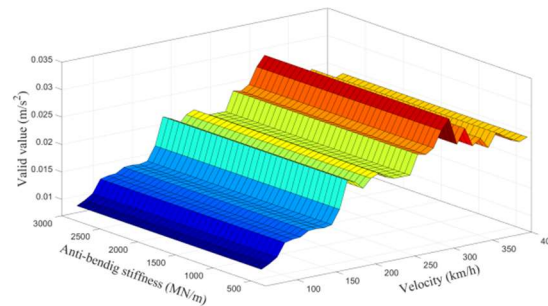


Figure13. Effective value of pitching mode vibration

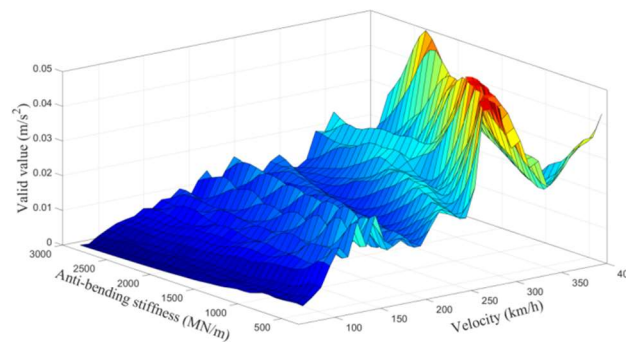


Figure 14. Effective value of vertical bending

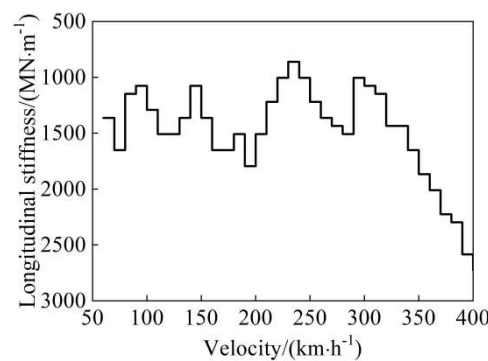


Figure 15. Critical velocity of stiffness attenuation vertical bending vibration

Figure 14 shows the trend of the effective value of the vertical bending mode vibration of the vehicle. It can be concluded that the vehicle's operating speed and the longitudinal stiffness of the anti-bending device have a significant impact on the vertical bending mode vibration of the vehicle. The effective value of the vertical bending vibration is closely related to vehicle speed and the stiffness of the anti-bending device. Figure 15 shows the critical velocity variation curve of the longitudinal stiffness of the anti-bending device on the attenuation of the vehicle's vertical bending modal vibration. It can be

seen that when the vehicle speed is below $350 \text{ km}\cdot\text{h}^{-1}$, the longitudinal stiffness of the anti-bending device begins to attenuate the vehicle's vertical bending vibration when it is between 1000 and $1800 \text{ MN}\cdot\text{m}^{-1}$. When the vehicle speed exceeds $350 \text{ km}\cdot\text{h}^{-1}$, the stiffness of the anti-bending device that attenuates the vehicle's vertical bending vibration gradually increases. The higher the speed, the greater the required stiffness. Figure 16 calculates the longitudinal stiffness of the anti-bending device on the vertical bending vibration attenuation trend of the vehicle in the main operating speed range ($250 \text{ km}\cdot\text{h}^{-1}$ - $350 \text{ km}\cdot\text{h}^{-1}$). It can be seen that the higher the speed, the higher the critical speed at which the anti-bending device attenuates the vertical bending vibration of the vehicle. When the stiffness value is less than $1800 \text{ MN}\cdot\text{m}^{-1}$, the anti-bending device only attenuates speeds below $350 \text{ km}\cdot\text{h}^{-1}$. When the stiffness value is higher than $1800 \text{ MN}\cdot\text{m}^{-1}$, the anti-bending device has a damping effect on the vertical bending vibration of the vehicle throughout the entire speed range, with a maximum attenuation of 54%.

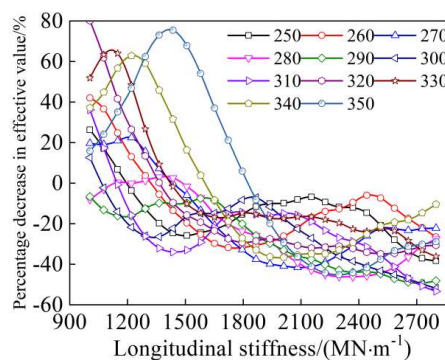


Figure 16. Attenuation trend of vertical bending vibration

5. Conclusion

This article studies the attenuation characteristics of a vehicle bending vibration suppression device on vehicle vibration. By mathematically modeling the vehicle system, the influence of the stiffness and installation position of the anti-bending device on the vertical bending modal frequency of the vehicle was analyzed. Combined with numerical solution methods, the influence characteristics of anti-bending devices on vehicle vibration response were studied. The corresponding relationship between the stiffness of anti-bending devices, vehicle comfort indicators, and third-order modal vibration was explored. The research conclusion is as follows:

- (1) The longitudinal stiffness of the anti-bending device can increase the vertical bending frequency of the vehicle by up to 70%. The closer the anti-bending device is to the middle of the vehicle, the higher the vertical bending modal frequency of the vehicle, and the maximum value is taken when it is in the middle of the vehicle.
- (2) The anti-bending device has a good suppression effect on the vertical bending vibration of the vehicle. Installing an anti-bending device can not only increase the vertical bending frequency of the vehicle, but also significantly attenuate the vibration amplitude of the vehicle at the vertical bending frequency.
- (3) The anti-bending device can significantly improve the comfort of the middle of the vehicle, especially affecting the comfort index of the middle of the vehicle in the range of $270 \text{ km}\cdot\text{h}^{-1}$ to $\text{km}\cdot\text{h}^{-1}$. The greater the longitudinal stiffness of the anti-bending device, the more the comfort index in the middle decreases (up to 6.7%). When the vehicle speed is around $300 \text{ km}\cdot\text{h}^{-1}$ and the stiffness is at least $\text{MN}\cdot\text{m}^{-1}$, the influence of the anti-bending device on the comfort of the vehicle center gradually becomes prominent.
- (4) The critical stiffness value of the anti-bending device that attenuates the vertical bending mode vibration of the vehicle when the vehicle speed is below $350 \text{ km}\cdot\text{h}^{-1}$ is concentrated in the range of 1000 - $1800 \text{ MN}\cdot\text{m}^{-1}$. The stiffness has a significant impact on the vertical bending modal vibration of

vehicles, and the higher the speed, the greater the damping effect on the vertical bending vibration of vehicles. The stiffness is greater than $1800 \text{ MN} \cdot \text{m}^{-1}$, and the device has a maximum attenuation of 54% for the vertical bending vibration of vehicles in the $250\text{-}350 \text{ km} \cdot \text{h}^{-1}$ range.

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