

# Investigation of Mechanical Resonance in a Damped Pendulum Using a Low-Cost, Classroom-Friendly Setup

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## Abstract

This report presents an investigation into the oscillatory behaviour of a simple pendulum under varying damping conditions and external driving forces, employing a low-cost, easy-to-operate, and visually accessible experimental setup designed for high school classroom demonstrations. The experiment explored both the transient response of free oscillations and the steady-state response under forced oscillations. Free oscillation trials were conducted in air, water, alcohol, and glycerol, representing negligible, light, moderate, and heavy damping regimes respectively. Damping effects were quantified by analysing amplitude decay, with characteristic exponential decay observed in light damping and rapid suppression of motion in heavy damping. Forced oscillations were induced through periodic driving, revealing distinct resonance behaviour under light and moderate damping, with resonance peaks shifting lower and broadening as damping increased. The amplitude response curves demonstrated qualitative agreement with the theoretical model of damped driven harmonic oscillators, showcasing key features such as amplitude suppression, phase lag development, and resonance peak broadening. This accessible and low-cost setup effectively illustrates fundamental oscillation phenomena, making it a suitable tool for classroom engagement and conceptual reinforcement of damping and resonance principles, with potential extensions to discussions on practical engineering applications such as shock absorption by tuned mass damper and structural resonance control.

## Keywords

**Simple Pendulum; Damping; Forced Oscillations; Resonance; Experimental Setup; High School Demonstration.**

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## 1. Introduction

Resonance is a ubiquitous and powerful phenomenon in physics, arising in any system capable of oscillation when driven at or near its natural frequency. In mechanical systems, resonance underpins the operation of devices ranging from clocks and musical instruments to seismographs and MEMS sensors. However, it can also lead to structural failure when oscillatory energy is unintentionally amplified, as in the well-documented collapse of the Tacoma Narrows Bridge in 1940, which remains a canonical example of resonance-induced mechanical failure [1]. Understanding how the amplitude and phase of an oscillating system respond to varying driving frequencies, and how damping influences these dynamics, is thus of both theoretical and practical importance.

The simple pendulum—one of the oldest and most fundamental systems in classical mechanics, extensively studied since the pioneering works of Galileo Galilei in the 17<sup>th</sup> century [2]—provides an accessible and illustrative platform for investigating oscillatory motion and mechanical resonance. It consists of a point mass suspended by a lightweight inextensible string, undergoing angular displacement under the influence of gravity, with a restoring force proportional to the sine of the displacement angle. Within the small-angle approximation, the simple pendulum exhibits near-ideal

simple harmonic motion, making it a cornerstone of introductory physics education [3]. Despite its inherent nonlinearity at larger amplitudes, the simplicity of construction, clear visualisation of oscillations, and direct physical interpretation of damping effects through fluid interaction render the simple pendulum highly suitable for classroom demonstrations. Moreover, its ability to exhibit both free and driven oscillations with adjustable damping provides a versatile framework for exploring key dynamical concepts such as energy dissipation, resonance, and amplitude response [4].

The aim of this experiment is to investigate the response of a simple pendulum to both free and forced oscillations under varying damping conditions. Specifically, the goals are to (i) determine the natural angular frequency  $\omega_0$  of the undamped system, (ii) quantify the exponential decay of amplitude in the transient response and compare the quality factor  $Q$  for different levels of damping, and (iii) analyse the frequency-dependent steady-state response under sinusoidal driving to extract resonant frequencies and amplitude response pattern to different driving frequencies.

In this experiment, a simple pendulum was subjected to both free and driven oscillations to investigate the effects of damping and mechanical resonance. The driving frequency was controlled via a variable-speed motor, while damping was introduced by submerging the pendulum bob in different liquids of varying viscosity, including water, 75% ethanol, and glycerol. Free oscillations were analysed using video tracking software to measure amplitude decay, allowing for the calculation of damping coefficients and quality factors. Forced oscillations were studied by manually measuring steady-state amplitudes across a range of driving frequencies to observe resonance behaviour and its dependence on damping.

The remainder of this report is organised as follows: Section 2 provides the theoretical framework underlying damped and driven harmonic motion, including derivations of the amplitude and phase response. Section 3 describes the experimental method and data acquisition procedures in detail. Section 4 presents the collected data. Section 5 interprets the results in light of the theoretical predictions, and Section 6 concludes with a summary of findings and their broader implications.

## 2. Theoretical Background

The motion of a simple pendulum subject to damping and external driving can be modelled as a damped, driven harmonic oscillator. At small angular displacement  $\theta(t)$ , small angle approximation applies, and the general equation of motion for angular displacement  $\theta(t)$  is given by:

$$I\ddot{\theta} + b\dot{\theta} + mgl\theta = A \cos(\omega t), \quad (1)$$

where  $I$  is the moment of inertia of the pendulum,  $b$  is the damping coefficient,  $A$  is the amplitude of the driving torque,  $l$  is the length of the string that connects the pivot and the bob, and  $\omega$  is the angular frequency of the driving force.

For a simple pendulum, assuming that the string is inextensible and weightless, the moment of inertia of the pendulum  $I$  is given by

$$I = ml^2 \quad (2)$$

The natural angular frequency of the simple pendulum is therefore given by

$$\omega_0 = \sqrt{\frac{g}{l}} \quad (3)$$

For a pendulum bob moving slowly through a fluid, where the velocity is low and the Reynolds number is small, the viscous damping force can be approximated by Stokes' law [5]:

$$F_{\text{damping}} = 6\pi\eta r v, \quad (4)$$

where  $\eta$  is the dynamic viscosity of the fluid (Pa·s),  $r$  is the radius of the spherical bob (m),  $v$  is the linear velocity of the bob (m/s).

In the case of a simple pendulum, the motion is rotational about the pivot point. For small angular displacements, the linear velocity at the bob is related to the angular velocity by

$$v = L \dot{\theta}, \quad (5)$$

where  $L$  is the length of the pendulum (m) and  $\dot{\theta}$  is the angular velocity (rad/s). Substituting into Stokes' law, the damping force becomes

$$F_{\text{damping}} = 6\pi\eta r L \dot{\theta}. \quad (6)$$

The corresponding damping torque  $T_{\text{damping}}$  is obtained by multiplying the damping force by the moment arm  $L$ :

$$T_{\text{damping}} = L \cdot F_{\text{damping}} = 6\pi\eta r L^2 \dot{\theta}. \quad (7)$$

Thus, the effective angular damping coefficient  $b$  is given by

$$b = 6\pi\eta r L^2, \quad (8)$$

which quantifies the viscous damping torque acting on the pendulum in terms of angular velocity.

By introducing the decay constant  $\gamma = \frac{b}{2I}$  and the natural angular frequency  $\omega_0$ , equation (1) can be rewritten in canonical form:

$$\ddot{\theta} + 2\gamma\dot{\theta} + \omega_0^2\theta = \frac{A}{I} \cos(\omega t). \quad (9)$$

This equation yields two components of the solution: a transient solution and a steady-state (forced) solution.

**Free (Undriven) Oscillations:** When  $A = 0$ , the homogeneous solution describes the free motion of the oscillator. In the case of light damping, where  $\gamma < \omega_0$ , the system exhibits oscillatory motion with an exponentially decaying amplitude. The general solution is given by

$$x(t) = x_0 e^{-\gamma t} \cos(\omega_1 t + \phi), \quad (10)$$

where  $\omega_1 = \sqrt{\omega_0^2 - \gamma^2}$  is the damped angular frequency, and  $\phi$  is the phase constant determined by initial

conditions. The oscillations decay gradually due to energy dissipation.

In the case of heavy damping, where  $\gamma > \omega_0$ , the motion is non-oscillatory and consists of two exponentially decaying terms. The general solution becomes

$$x(t) = Ae^{-(\gamma + \sqrt{\gamma^2 - \omega_0^2})t} + Be^{-(\gamma - \sqrt{\gamma^2 - \omega_0^2})t}, \quad (11)$$

where A and B are constants determined by initial conditions. The system returns to equilibrium without oscillating, characterised by a slow, monotonic decay.

The quality factor Q, a dimensionless measure of the sharpness of resonance or the rate of energy loss, is defined by:

$$Q = \frac{\omega_0}{2\gamma}. \quad (12)$$

Alternatively, using the time  $t_e$  for the amplitude to decay by a factor of e, Q can be estimated by:

$$Q = \frac{\omega_0 t_e}{2}. \quad (13)$$

Forced Oscillations: For nonzero driving torque, the system eventually reaches a steady-state oscillation at the driving frequency  $\omega$ . The steady-state amplitude of oscillation is given by:

$$X(\omega) = \frac{A/I}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}}, \quad (14)$$

and the phase lag  $\phi$  between the driving force and the response is:

$$\tan \phi = \frac{2\gamma\omega}{\omega_0^2 - \omega^2}. \quad (15)$$

The amplitude reaches a maximum at the resonant frequency  $\omega_{\max}$ , given by:

$$\omega_{\max}^2 = \omega_0^2 - 2\gamma^2. \quad (16)$$

In the limit of low damping ( $\gamma \ll \omega_0$ ), the resonance condition simplifies to  $\omega_{\max} \approx \omega_0$ , and the amplitude at resonance becomes:

$$X(\omega_{\max}) \approx \frac{A}{2I\gamma\omega_0} \quad (17)$$

This relation shows that for low damping case, the resonant amplitude decreases as decay constant  $\gamma$  increases.

### 3. Method

#### 3.1 Apparatus and Setup

The apparatus used to investigate mechanical resonance consisted of a simple pendulum fixed on a mobile platform, which is periodically driven by a motor via a drive wheel and push rod mechanism. The driving frequency of the motor was adjusted by rotating a speed control screw, allowing continuous variation of the input frequency. Damping was provided by submerging the pendulum bob into various types of liquid with different viscosity in a large sink mounted on a stationary platform. For free oscillations without external driving torque, angular displacement over time was measured by analysing recorded video footage with Tracker [6], an open-source video analysis software designed for tracking motion in physics experiments. For forced oscillation, a protractor was fixed to the pivot. The angular amplitude at each driving frequency was manually measured by tracking the maximum angular displacement at steady state.

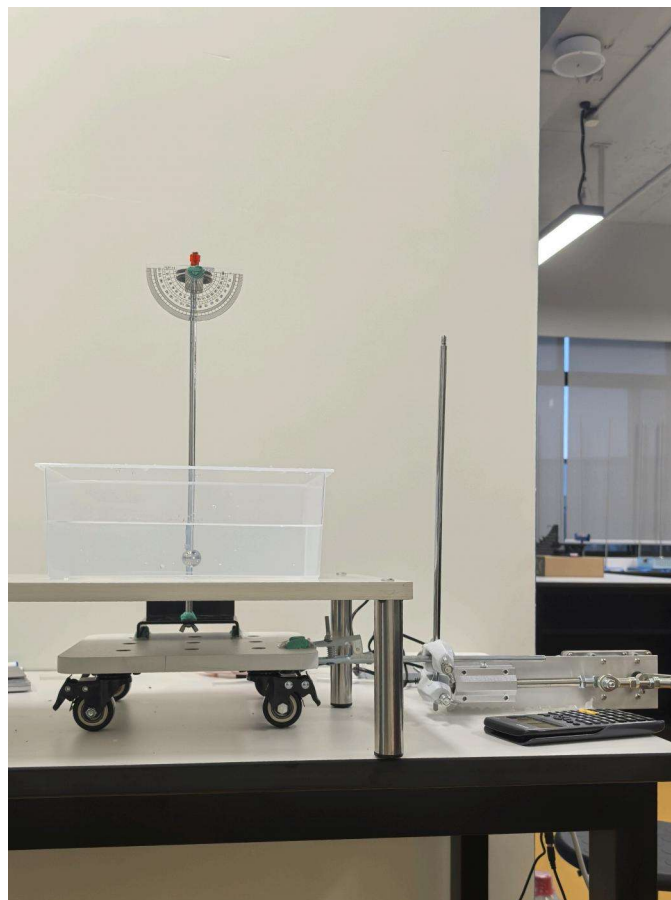


Figure 1. Experimental setup.

#### 3.2 Free Oscillation Measurement

To study free oscillations, the pendulum was released from rest without any external driving force, at a small initial angular displacement ( $\theta_0 \leq 10^\circ$ ) to ensure the validity of the small-angle approximation, where  $\sin \theta \approx \theta$  holds to within less than 1% error [3].

In the light damping case, water and 75% aqueous ethanol solution by volume was used as a damping medium. In the heavy damping case, glycerol was used. The container was fixed in place on the stationary platform, and the motion of the pendulum was recorded on video. The auto-tracking function of the tracker followed the pendulum bob and generated displacement time data. The decay constant  $\gamma$  was determined from the graph, and the damping coefficient  $b$  and the quality factor  $Q$  were calculated and compared with theory.

### 3.3 Driven Oscillation Measurement

For the driven oscillation study, the motor was activated to supply periodic external driving torque. The angular driving frequency of the motor was measured using a stopwatch by timing one complete revolution. By adjusting the motor speed and observing the resulting angular displacement with a protractor, an amplitude-driving frequency graph was generated.

Forced oscillation with negligible damping was first studied by operating the system without any liquid in the sink, thereby minimising resistive forces. As the driving frequency approached the natural frequency of the pendulum from both sides, the induced amplitude increased progressively. Beyond a certain threshold, the oscillations exhibited visible instability characterised by large-amplitude swinging, noticeable irregularities in trajectory, and occasional lateral or out-of-plane motion. In this regime, the simple harmonic approximation breaks down, and the pendulum's response becomes highly sensitive to small perturbations, resulting in erratic or asymmetric motion that can lead to collisions with the support structure. At these thresholds, the resultant amplitude is recorded as infinity.

After the natural frequency range was identified, the pendulum bob was submerged to different types of damping liquid. The response of the system under different damping conditions was then examined with different driving frequencies, focusing on the amplitude of the oscillation. By gradually varying the angular velocity of the driving system, the pendulum exhibited different levels of oscillation. A significantly increased amplitude indicated that the driving frequency was approaching the resonant frequency. The observed resonance behaviour was then compared with the theoretically predicted natural frequency to assess the consistency of the experimental results with the theoretical model.

## 4. Results

### 4.1 Natural Frequency Calculation and Measurement

The mass of the bob was measured to be  $0.03139 \pm 0.00001$  kg by an electronic mass balance. The length of the pendulum string was set to  $L = 0.3$  m, from which the theoretical natural angular frequency was calculated using equation (3), giving  $\omega_0 = 5.71$  rad/s. To experimentally verify this value, a stopwatch was used to measure the time taken for 10 complete oscillations. The average period was found to be approximately  $1.11 \pm 0.01$  s, corresponding to an experimental angular frequency of  $\omega = \frac{2\pi}{T} = 5.66 \pm 0.05$  rad/s, which shows excellent agreement with the theoretical prediction. Expressed in revolutions per minute (rpm), the angular frequency corresponds to  $54.1 \pm 0.5$  rpm.

### 4.2 Free Oscillation Measurement

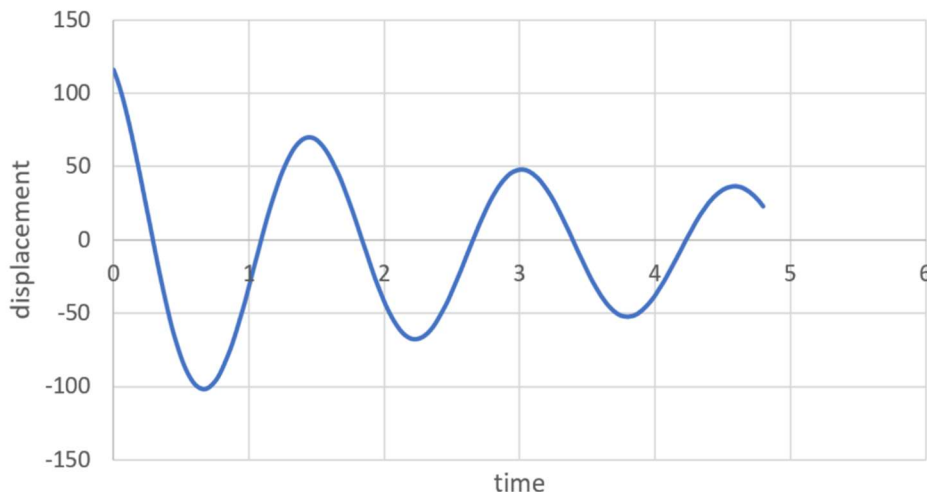
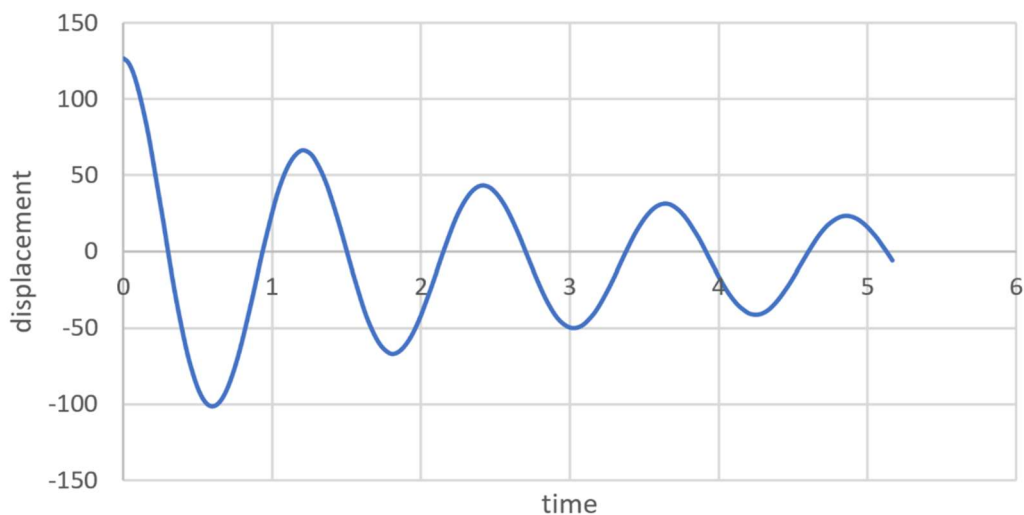
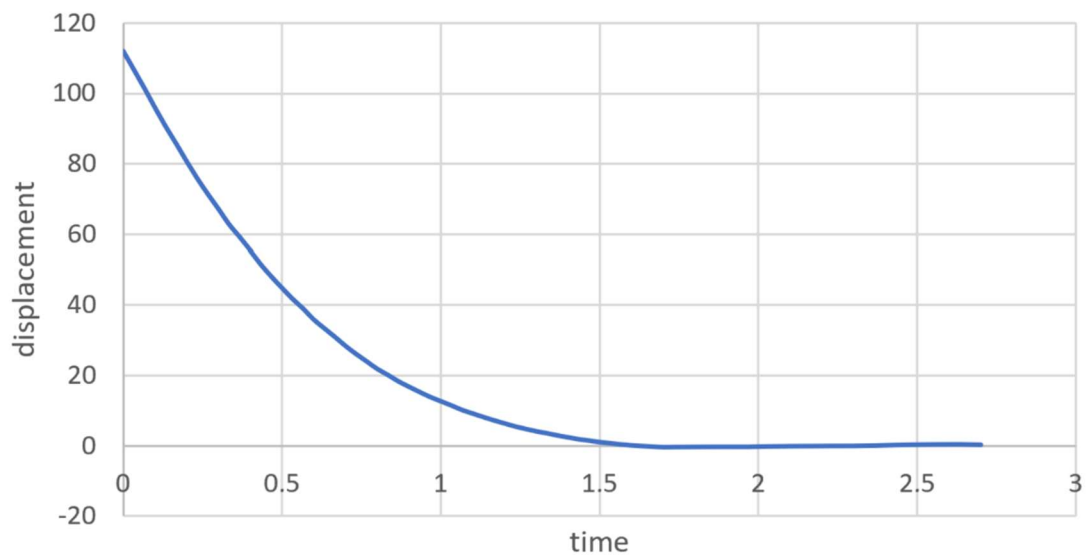


Figure 2. Displacement-time graph of water



**Figure 3.** Displacement-time graph of alcohol



**Figure 4.** Displacement-time graph of glycerol

### 4.3 Driven Oscillation Measurement

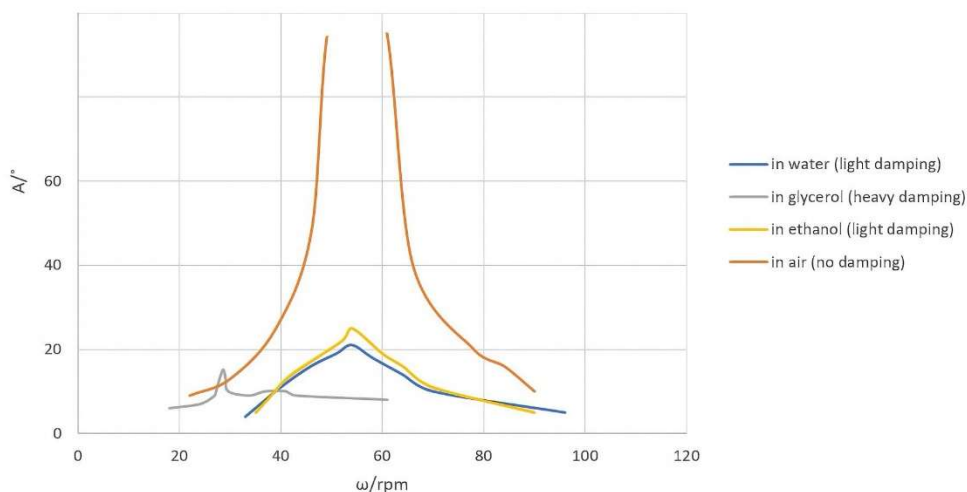
**Table 1.** Data measured and used to plot amplitude-driving frequency graph

in air (no damping)	
$\omega$ /rpm	$A/^\circ$
22	9
30	13
39	25
46	48
50	$\infty$
60	$\infty$
66	40
78	20
84	16
90	10

in ethanol (light damping)	
$\omega/\text{rpm}$	$A/^\circ$
35	5
38	9
42	14
52	22
54	25
60	19
64	16
70	11
90	5

in water (light damping)	
$\omega/\text{rpm}$	$A/^\circ$
33	4
40	11
46	16
51	19
54	21
58	18
64	14
70	10
96	5

in glycerol (heavy damping)	
$\omega/\text{rpm}$	$A/^\circ$
18	6
24.1	7
27	9
27	9
28.5	15
29.1	13
29.6	10
33.7	9
37	10
41	10
43	9
61	8



**Figure 5.** Driven oscillation Amplitude-angular displacement graph

## 5. Discussion

### 5.1 Measured Values and Theoretical Agreement

The experimental results show good agreement with the theoretical model of a damped, driven harmonic oscillator. The natural angular frequency of the undamped pendulum was measured to be  $\omega_0 = 5.66 \pm 0.05$  rad/s, or equivalently  $54.1 \pm 0.5$  rpm. This value is further supported by the amplitude–frequency response curves obtained for water and ethanol, which show resonant frequencies of  $54 \pm 2$  rpm and  $54 \pm 3$  rpm respectively. These results are consistent with the theoretical expression given in Equation (16), which predicts that for small damping coefficients  $b$  as in the cases of water and ethanol the decay constant  $\gamma$  is small, and the resonant frequency remains close to the natural frequency.

In both water and alcohol, the displacement-time graphs exhibited exponentially decaying sinusoidal curves. Compared to water, alcohol resulted in a faster decrease in amplitude, allowing the pendulum to oscillate for a shorter period. Fitting the enveloping curves of figure 2 and 3 to exponential decay model shows that the decay constant for water and 75% ethanol were  $0.238 \text{ s}^{-1}$  and  $0.332 \text{ s}^{-1}$  respectively. This is expected as alcohol has a higher dynamic viscosity than water [7], which leads to higher damping coefficient  $b$  as predicted by equation (8), and therefore a higher decay constant for alcohol. Equation (12) shows that the quality factor decreases as degree of damping increases for alcohol as compared to water, indicating the oscillator becomes worse.

When the container was filled with glycerol, the system exhibited heavy damping, causing the pendulum bob to return rapidly to its equilibrium position without oscillating. The corresponding displacement-time curve quickly approached zero, reflecting the overdamped behaviour. Fitting the curve in Figure 4 with the heavy damping model (11), we can calculate the decay constant to be as high as  $8.31 \text{ s}^{-1}$ .

The amplitude–frequency response curves (Figure 6) closely follow the theoretical form described by Equation (14). It is clearly observable that as damping increases from water and 75% ethanol to glycerol,

the resonant peak decreases in amplitude, broadens, and shifts to a lower frequency, in agreement with theoretical predictions.

### 5.2 Sources of Error

Interestingly, despite our earlier findings that 75% ethanol introduces greater damping than water in free oscillations, consistent with its higher dynamic viscosity, the driven oscillation data shown in Figure 6 reveals the opposite trend. Specifically, the amplitude at resonance remains higher in 75% ethanol than in water. This could be attributed to the higher density of water that increases the effective fluid inertia acting on the pendulum, which is more influential in driven oscillations where continuous energy input amplifies the role of inertial drag and invalidates Stoke's Law(4).

Despite the overall agreement, several sources of experimental uncertainty may have affected the precision of the results. In forced oscillation measurement, the angular displacement readings relied on visual reading of instantaneous maximum angle of a moving protractor, introducing a typical uncertainty of approximately  $\pm 2$  in amplitude measurements. Manual timing period of driving motor with a stopwatch introduced human reaction time error, particularly in determining periods for high-frequency oscillations.

One significant source of error in the experiment arises from the inevitable vertical oscillations of the pendulum bob, caused by slight imperfections in the driving mechanism and fluid interactions. These vertical motions introduce additional forces and energy dissipation mechanisms not accounted for in the simple two-dimensional theoretical model, potentially leading to overestimated damping effects and irregular amplitude measurements, particularly in the driven oscillation setup.

Another source of uncertainty arises from the release mechanism used during free oscillation measurements. The pendulum was released manually by hand, which may have inadvertently

introduced a small initial torque or lateral impulse. This could lead to non-uniform initial conditions between trials, particularly affecting the early cycles used in amplitude decay analysis, leading to error in decay constant measurement and quality factor calculation.

Additional unmodelled factors such as friction in the pivot bearings, asymmetry in the driving arm, or variations in damping due to liquid current may also have introduced systematic deviations from idealised behaviour.

### 5.3 Suggestions for Improvement

Several improvements could enhance the accuracy and repeatability of the experiment. First, replacing the manual stopwatch with a digital timing system or angular position sensor would reduce human error and increase timing precision. Second, using optical measurement techniques (e.g., a laser pointer and scale) would improve amplitude resolution.

A possible method to reduce vertical oscillation is the incorporation of a guide ring positioned just above the damping liquid. The guide ring acts as a low-friction constraint around the pendulum rod or string, allowing free angular oscillation while suppressing unwanted vertical motion. By limiting the vertical displacement of the pendulum bob, this setup minimises the introduction of additional energy dissipation mechanisms not accounted for in the theoretical model. Care must be taken to ensure that the guide ring does not significantly interfere with the natural swinging motion, for instance by maintaining a sufficient clearance to avoid contact during regular oscillations. This modification would improve the accuracy of damping measurements by reducing disturbances caused by fluid drag and mechanical vibrations..

To reduce variability in initial conditions, a mechanical release mechanism, such as a spring-loaded latch or electromagnetic clamp, could be used to ensure consistent, force-free release of the pendulum from rest. This would minimise the introduction of spurious torques and improve the reproducibility of amplitude decay measurements.

Repeating each measurement multiple times and computing mean values with standard deviations would also improve the statistical robustness of the data and help identify any outliers or anomalies.

## 6. Conclusion

This experiment successfully demonstrated the resonance behaviour of a simple pendulum under varying degrees of damping and external driving, in agreement with the theoretical model of a damped, driven harmonic oscillator. As damping increased from water and 75% ethanol to glycerol, the quality factor  $Q$  and resonant amplitude both decreased, while the resonance peak broadened and shifted to lower frequencies. These observations quantitatively confirmed the predicted relationships between damping strength and resonance characteristics. Such findings are directly applicable to real-world engineering systems, particularly in the design of tuned mass dampers (TMD), where controlled damping is crucial for reducing structural vibrations in bridges and skyscrapers, including examples like Taipei 101. Understanding how damping modifies resonant behaviour offers valuable insight into achieving both stability and resilience in mechanical structures.

In addition, the experimental setup employed in this study was deliberately designed to be low-cost, visually accessible, and straightforward to operate, making it well-suited for high school physics classrooms. The combination of simple mechanical components, clear visualisation of oscillatory motion, and straightforward data collection methods provides an effective teaching tool to introduce students to key physical concepts such as natural frequency, damping, resonance, and energy dissipation, thereby bridging theoretical understanding with practical observation.

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