Fuzzy Similarity based on Subtraction Set Pair Potential and its Applications

Zihang Zhao, Hongkui Hu*

North China University of Science and Technology, Tangshan 063210, China

Abstract

To address the issue where existing intuitionistic fuzzy set similarity measurement methods struggle to distinguish samples in several special data cases, a novel intuitionistic fuzzy similarity calculation method is proposed by delving into the dynamic characteristics among membership, hesitation, and non - membership degrees, and incorporating the subtraction set pair potential from the set pair analysis theory. The basic properties of intuitionistic fuzzy similarity are verified for this method. By calculating several special data cases to test its discriminability and comparing it with multiple existing measurement methods, results show that it effectively overcomes the problem of indistinguishable samples in special data cases. Furthermore, the traditional TOPSIS decision - making model is improved by presenting a pre - scoring TOPSIS model that integrates the subtraction set pair potential. Finally, the new intuitionistic fuzzy set similarity calculation method is combined with this model and applied to multi - attribute decision - making, demonstrating its reliability and effectiveness.

Keywords

Intuitionistic Fuzzy Set; Similarity Measure; Connection Number; Subtraction Set Pair Potential; Multi-attribute Decision Making.

1. Introduction

In 1965, Zadeh introduced fuzzy set theory[1], providing an effective mathematical framework for addressing issues of vagueness and uncertainty. As an extension of fuzzy set theory, Atanassov proposed the theory of intuitionistic fuzzy sets[2-3] in 1986, which characterizes fuzziness through membership, non-membership, and hesitation degrees. This approach offers greater flexibility and practicality in handling uncertain information compared to traditional fuzzy sets, and has been widely applied in fields such as decision analysis[4], pattern recognition[5], and fuzzy clustering[6].

Recently, the similarity measurement of intuitionistic fuzzy sets has garnered extensive scholarly attention. Chen[7-8] developed a similarity measure based on a scoring function for vague sets, while Bustince and Burillo[9] demonstrated the equivalence between vague sets and intuitionistic fuzzy sets. Hong and Kim[10] provided examples illustrating the limitations of Chen's method in practical applications and proposed improvements. Li Fan et al[11]. identified discrepancies between Chen's and Hong's similarity measures and actual scenarios, proposing new methods to address these issues. Chen and Randyanto[12] employed the median interval principle for similarity assessment, whereas Hung and Yang[13] defined similarity using the Hausdorff distance. Hwang and Yang[14] introduced a similarity measure based on the total expected difference between two intuitionistic fuzzy sets, utilizing Sugeno integrals. Yang Yong et al[15]. proposed a similarity measure employing sine functions, and Chen and Chang[16] defined a measure based on the area difference and similarity on the x-axis of the sets. Liang Meishe et al[17]. further incorporated the semantic implications of hesitation degrees to mitigate counterintuitive results in similarity calculations. Although the development of similarity measures for intuitionistic fuzzy sets has progressed rapidly, challenges

ISSN: 2414-1895 DOI: 10.6919/ICJE.202510_11(10).0011

remain in distinguishing certain data samples, and the general applicability and computational accuracy of these measures require further optimization.

Given this, the present study considers the dynamic characteristics among membership degree, non-membership degree, and hesitation degree. It introduces the concept of the difference set potential from the theory of soft set pair analysis, proposing a novel similarity measure for intuitionistic fuzzy sets. The fundamental properties of this measure are rigorously demonstrated. Furthermore, the traditional TOPSIS method is improved and integrated with the newly proposed similarity measure to address multi-attribute decision-making problems. The validity and effectiveness of the approach are verified through case studies.

2. Preliminaries

2.1 Intuitionistic Fuzzy Sets

Intuitionistic fuzzy sets extend traditional fuzzy sets by accounting not only for the membership degree of elements within a set but also for their non-membership degree, providing a more comprehensive representation of uncertainty.

Definition 2.1: Let $X=\{x_1,x_2,\ldots,x_n\}$ be a non-empty finite domain, and an intuitive fuzzy set on X can be defined as:

$$A(x)=\{\langle x,u_A(x),v_A(x)\rangle \mid x\in X\} \tag{1}$$

Where u_A represents the membership degree of element x to set A, satisfying condition $u_A\in[0,1]$, and v_A denotes the non-membership degree of element x to set A, satisfying condition $0\le\mu_A(x)+v_A(x)\le 1$. Additionally, $\forall x\in U$ satisfies condition $0\le\mu_A(x)+v_A(x)\le 1$. $1-\mu_A(x)-v_A(x)$ is defined as the hesitation degree of x with respect to A, denoted as $\pi_A(x)$.

2.2 Similarity Measure of Intuitionistic Fuzzy Sets

Definition 2.2: For any intuitionistic fuzzy set $A,B,C\in IFS(X)$, define the membership function $S:IFS(X)\times IFS(X)\to[0,1]$, which satisfies the following conditions:

(1) $0\le S(A,B)\le 1$;

(2) $S(A,B)=1$ if and only if two $A=B$

(3) $S(A,B)=S(B,A)$;

(4) If $A\subseteq B\subseteq C$, then $S(A,C)\le S(A,B)$ and $S(A,C)\le S(B,C)$.

The similarity between S and $IFS(X)$.

2.3 Triadic Contact Number and the Potential of Subtraction Set Pairs

Set Pair Analysis (SPA)[18], established by Chinese scholar Zhao Keqin, posits that uncertainty is an inherent and universal characteristic of phenomena. Any entity or system is viewed as a dialectical unity comprising both certainty and uncertainty. The core principle of this theory is to analyze the research object’s deterministic and indeterminate relationships as an integrated uncertain system. Within this framework, certainty encompasses the dimensions of "identity" and "opposition," while uncertainty specifically pertains to "difference." SPA examines the system through the lenses of sameness, difference, and opposition, which are interconnected, mutually influential, and mutually constraining, with the potential for transformation under certain conditions. The triadic contact number serves as a fundamental analytical tool, providing a comprehensive representation of the system’s developmental trends from the perspectives of similarity, difference, and opposition.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202510_11(10).0011

Definition 2.3: The general form of the triadic contact number is as follows:

$$\mu = a + bi + cj \tag{2}$$

Where a represents the degree of similarity, b denotes the degree of divergence, and c indicates the degree of opposition; all are non-negative real numbers satisfying the conditions of the divergence degree ($a+b+c=1$). $i \in [-1,1]$ is the divergence coefficient, and the opposition coefficient $j=-1$.

The subtraction set pair potential[19] is a type of conjugate function of the triadic correlation coefficient, employed to characterize the relative certainty state and developmental trend of the studied set pair events within the current macro-expectation hierarchy.

Definition 2.4: The general form of the difference set potential function:

$$s_f(u) = a - c + ba - bc = (a-c)(1+b) \tag{3}$$

Wherein, the satisfaction of $s_f(u) \in [-1,1]$ is determined by calculating the disparity ratio; the values of the difference set for potential are ordered based on the magnitude of $s_f(u)$. According to the "Equal Division Principle," $s_f(u)$ is segmented into five potential levels: inverse potential $s_f(u) \in [-1.0,-0.6)$, partial inverse potential $s_f(u) \in [-0.6,-0.2)$, neutral potential $s_f(u) \in [-0.2,0.2]$, partial same potential $s_f(u) \in (0.2,0.6]$, and same potential $s_f(u) \in (0.6,1.0]$.

3. Similarity Measure Formula for Intuitionistic Fuzzy Sets (SPP-IFS)

According to the theory of set pair analysis, the essence of the subtraction set pair potential lies in representing the relative certainty state and developmental trend of the research object expressed by the connection degree within the current macro-expectation level. Existing similarity calculation formulas for intuitionistic fuzzy sets encounter difficulties in distinguishing certain types of data, necessitating further exploration of the intrinsic information of intuitionistic fuzzy sets to enhance computational accuracy. Intuitionistic fuzzy sets express fuzziness through membership degree, non-membership degree, and hesitation degree, with the relative certainty state and developmental trend also serving as influencing factors for the similarity between sets. Currently, Liu Xiumei [20] has transformed intuitionistic fuzzy sets into connection degrees, providing theoretical support for integrating set pair analysis concepts and redefining the similarity calculation formula for intuitionistic fuzzy sets. The similarity measure for intuitionistic fuzzy sets (SPP-IFS) is defined as follows:

$$S_{SPP}(A,B) = 1 - \frac{1}{8}\left\{\left|u_A(x) - u_B(x) + v_B(x) - v_A(x)\right| + \left|u_A(x) - u_B(x)\right| + \left|v_A(x) - v_B(x)\right| + 2\left|s_f(A) - s_f(B)\right|\right\} \tag{4}$$

Where $s_f(A)$ and $s_f(B)$ represent the difference set potentials of the corresponding contact numbers in the intuitionistic fuzzy sets, specifically $s_f(A) \in [-1,1]$ and $s_f(B) \in [-1,1]$.

3.1 Proof of the Similarity Formula between SPP-IFS

The following demonstrates that the four conditions satisfied by the similarity measure of SPP-IFS are met:

(1)It is evident that since $S_{SPP}(A,B) \le 1$ holds, the satisfaction of $\left|u_A(x) - u_B(x) + v_B(x) - v_A(x)\right| \le 2$ implies that $\left|u_A(x) - u_B(x)\right| \le 1$, $\left|v_A(x) - v_B(x)\right| \le 1$, and $\left|s_f(A) - s_f(B)\right| \le 2$ are also satisfied, thereby leading to $S_{SPP}(A,B) \ge 1 - \frac{2+1+1}{8} + \frac{1}{2} = 0$.

(2)Necessity: Given $S_{SPP}(A,B)=1$, then $|u_A(x)-u_B(x)|=0$, then $|v_A(x)-v_B(x)|=0$, from which $u_A(x)=u_B(x)$ and $v_A(x)=v_B(x)$ are readily derived, leading to $A=B$. Sufficiency: Given $A=B$, then $s_f(A)-s_f(B)=0$, then $|u_A(x)-u_B(x)+v_B(x)-v_A(x)|=0$, then $|u_A(x)-u_B(x)|=0$, then $|v_A(x)-v_B(x)|=0$, culminating in $S_{SPP}(A,B)=1-0-0=1$.

(3)Easily obtained $|u_A(x)-u_B(x)+v_B(x)-v_A(x)|=|u_B(x)-u_A(x)+v_A(x)-v_B(x)|$ $|u_A(x)-u_B(x)|=|u_B(x)-u_A(x)|$, then $|u_A(x)-u_B(x)|=|u_B(x)-u_A(x)|$, then $|v_A(x)-v_B(x)|$, then $s_f(A)=s_f(B)=0$, and consequently $S_{SPP}(A,B)=S_{SPP}(B,A)$.

(4)Based on $A\subseteq B\subseteq C$, it can be inferred that $u_A(x)\ge u_B(x)\ge u_C(x)$ and $v_A(x)\le v_B(x)\le v_C(x)$. From the properties of the difference set and the order, $s_f(A)\le s_f(B)\le s_f(C)$ can be deduced, leading to the determination of $|s_f(A)-s_f(C)|\ge|s_f(B)-s_f(C)|$; similarly, $|s_f(A)-s_f(B)|\ge|s_f(A)-s_f(C)|$ can be derived. Consequently, $|u_A(x)-u_B(x)+v_B(x)-v_A(x)|\le|u_A(x)-u_C(x)+v_C(x)-v_A(x)|$, $|u_A(x)-u_B(x)|\le|u_A(x)-u_C(x)|$, and $|v_A(x)-v_B(x)|\le|v_A(x)-v_C(x)|$ can be obtained, which in turn allow the deduction of $|u_B(x)-u_C(x)+v_C(x)-v_B(x)|\le|u_A(x)-u_C(x)+v_C(x)-v_A(x)|$, $|u_B(x)-u_C(x)|\le|u_A(x)-u_C(x)|$, and $|v_B(x)-v_C(x)|\le|v_A(x)-v_C(x)|$, indicating that $S_{SPP}(A,C)\le S_{SPP}(A,B)$ and $S_{SPP}(A,C)\le S_{SPP}(B,C)$ are present.

3.2 Definition of the Weighted SPP-IFS Similarity Formula

Definition 3.2: Let $A(x)=\{\langle x,u_A(x),v_A(x)\rangle \mid x\in X\}$ and $B(x)=\{\langle x,u_B(x),v_B(x)\rangle \mid x\in X\}$ be two intuitionistic fuzzy sets on the domain X, with $X=\{x_1,\dots,x_n\}$. The weighted SPP-IFS similarity measure between A and B is given by formula S_{SPP} as follows:

$$S_{SPP}(A(x_i),B(x_i))=$$
$$1-\frac{1}{8}\sum_{i=1}^{n} w_i\left\{|u_A(x_i)-u_B(x_i)+v_B(x_i)-v_A(x_i)|+|u_A(x_i)-u_B(x_i)|+|v_A(x_i)-v_B(x_i)|+2|s_f(A(x_i))-s_f(B(x_i))|\right\} \tag{5}$$

Where $w_i\in[0,1]$ represents the weight, and with $\sum_{i=1}^{n} w_i=1$, the proof of the similarity formula is analogous to that of Equation 2.1.

4. Comparative Analysis

This section employs the SPP-IFS similarity measure to process five categories of special data from reference [21]. The data and some methods are calculated and rounded to four decimal places. A comparative analysis is conducted against the listed methods, with measurement results used to demonstrate that the proposed approach maintains good discriminative ability when handling both these special data types and conventional data. This underscores the method's versatility and rationality.

Example 4-1: The numbers C_1, C_2, and C_3 represent three types of intuitive fuzzy patterns $P_1, P_2 a$ and P_3 defined over the domain $X=\{x_1,x_2,x_3\}$. Q denotes an unknown pattern classified as one of the modes C_1, C_2 or C_3, as illustrated below.

$$C_1=\{\langle x_1,1,0\rangle,\langle x_2,0.8,0\rangle,\langle x_3,0.7,0.1\rangle\}$$

$$C_2=\{\langle x_1,0.8,0.1\rangle,\langle x_2,1,0\rangle,\langle x_3,0.9,0\rangle\}$$

ISSN: 2414-1895 DOI: 10.6919/ICJE.202510_11(10).0011

$$C_3=\{\langle x_1,0.6,0.2\rangle,\langle x_2,0.8,0\rangle,\langle x_3,1,0\rangle\}$$

$$Q=\{\langle x_1,0.5,0.3\rangle,\langle x_2,0.6,0.2\rangle,\langle x_3,0.8,0.1\rangle\}$$

Comparison of similarity results obtained through different calculation methods, as presented in Table 1.

Table 1. Comparison of similarity results obtained through different computational methods

	$S_*(C_1,Q)$	$S_*(C_2,Q)$	$S_*(C_3,Q)$	**Classification outcomes**
S_C	0.7833	0.7833	0.8500	P_3
S_{HK}	0.7833	0.7833	0.8500	P_3
S_{LX}	0.7833	0.7833	0.8500	P_3
S_{LO}	0.7323	0.7585	0.8419	P_3
S_{DC}	0.7833	0.7833	0.8500	P_3
S_M	0.7833	0.7833	0.8500	P_3
S_{LS1}	0.7833	0.7833	0.8500	P_3
S_{LS2}	0.7833	0.7833	0.8500	P_3
S_{LS3}	0.8389	0.8389	0.8944	P_3
S_{HY1}	0.7333	0.7333	0.8333	P_3
S_{HY2}	0.6200	0.6297	0.7571	P_3
S_{HY3}	0.5789	0.5789	0.7430	P_3
S_Y	0.9353	0.9519	0.9724	P_3
S_{ZY}	N/A	N/A	N/A	Indeterminate
S_{CR}	0.7591	0.7699	0.8471	P_3
S_{BA}	0.7833	0.7833	0.8500	P_3
WSN_1[15]	0.4763	0.4549	0.5951	P_3
WSN_2[15]	0.4763	0.4549	0.5951	P_3
S_{LM}[17]	0.7208	0.6375	0.8042	P_3
S_L[11]	0.7833	0.7833	0.8500	P_3
S_{SPP}	0.6917	0.6900	0.7433	P_3

The similarity calculation results are presented in Table 1. The similarity rankings for items $S_{SPP}(C_1,Q)=0.6917$, $S_{SPP}(C_2,Q)=0.6900$, and $S_{SPP}(C_3,Q)=0.7433$ are as follows: $S_{SPP}(C_3,Q)>S_{SPP}(C_1,Q)>S_{SPP}(C_2,Q)$ indicating that item P_3 is most similar to item Q.

Analysis: The dataset exhibits significant variations in membership degree, non-membership degree, and hesitation degree, which aligns with the characteristics observed in most data sets. This demonstrates that the proposed method has good discriminative capability on conventional datasets. However, it is not possible to determine the specific rankings of S_C, S_{HK}, S_{LX}, S_{LO}, S_{DC}, S_M, S_{LS1}, S_{LS2}, S_{LS3}, S_{HY1}, S_{HY2}, S_{HY3}, S_{BA} and S_L, although the similarity between P_3 and Q is the highest, the method cannot further distinguish the similarity relationships among P_1, P_2 and Q.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202510_11(10).0011

The results obtained using the proposed approach are consistent with most existing computational outcomes and additionally enable the identification of the similarity between P_1, P_2 and Q, indicating that this method possesses high versatility in practical problem-solving scenarios.

Example 4-2: The numbers C_1, C_2, and C_3 represent three types of intuitive fuzzy patterns $P_1, P_2 a$ and P_3 defined over the domain $X=\{x_1,x_2,x_3\}$. Q denotes an unknown pattern classified as one of the modes C_1, C_2 or C_3, as illustrated below.

$$C_1=\{\langle x_1,0.1,0.1\rangle,\langle x_2,0.5,0.1\rangle,\langle x_3,0.1,0.9\rangle\}$$

$$C_1=\{\langle x_1,0.1,0.1\rangle,\langle x_2,0.5,0.1\rangle,\langle x_3,0.1,0.9\rangle\}$$

$$C_3=\{\langle x_1,0.7,0.2\rangle,\langle x_2,0.1,0.8\rangle,\langle x_3,0.4,0.4\rangle\}$$

$$Q=\{\langle x_1,0.4,0.4\rangle,\langle x_2,0.6,0.2\rangle,\langle x_3,0,0.8\rangle\}$$

Table 2. Comparison of similarity results obtained through different computational methods

	$S_*(C_1,Q)$	$S_*(C_2,Q)$	$S_*(C_3,Q)$	**Classification outcomes**
S_C	1.0000	1.0000	0.6000	Indeterminate
S_{HK}	0.8333	0.9333	0.6000	P_2
S_{LX}	0.9167	0.9667	0.6000	P_2
S_{LO}	0.8085	0.9184	0.5797	P_2
S_{DC}	1.000	1.0000	0.6000	Indeterminate
S_M	0.8333	0.9333	0.6000	P_2
S_{LS1}	0.8333	0.9333	0.6000	P_2
S_{LS2}	0.9167	0.9667	0.6000	P_2
S_{LS3}	0.8889	0.9556	0.7222	P_2
S_{HY1}	0.8333	0.9333	0.5667	P_2
S_{HY2}	0.7571	0.898	0.4437	P_2
S_{HY3}	0.7143	0.8750	0.3954	P_2
S_Y	0.9954	0.9989	0.6709	P_2
S_{ZY}	N/A	N/A	0.5167	Indeterminate
S_{CR}	0.9548	0.9883	0.5883	P_2
S_{BA}	0.9444	0.9778	0.6383	P_2
WSN_1[15]	0.3871	0.6852	0.2469	P_2
WSN_2[15]	0.3871	0.6852	0.2469	P_2
S_{LM}[17]	0.7583	0.9000	0.5417	P_2
S_L[11]	0.9167	0.9667	0.6000	P_2
S_{SPP}	0.8800	0.8933	0.5800	P_2

ISSN: 2414-1895 DOI: 10.6919/ICJE.202510_11(10).0011

Comparison of similarity results obtained through different calculation methods, as presented in Table 2.

The similarity calculation results are presented in Table 2. The similarity rankings for items $S_{SPP}(C_1,Q)=0.8800$, $S_{SPP}(C_2,Q)=0.8933$, and $S_{SPP}(C_3,Q)=0.5800$ are as follows: $S_{SPP}(C_2,Q)>S_{SPP}(C_1,Q)>S_{SPP}(C_3,Q)$ indicating that item P_2 is most similar to item Q.

Analysis: This dataset is an extreme case, where in the intuitionistic fuzzy numbers, the membership and non-membership degrees are equal for some elements. As shown in Table 2, the results indicate that elements S_C, S_{DC} and S_{ZY} cannot be definitively determined. For elements S_C and S_{DC}, the calculated intuitionistic fuzzy sets yield a similarity of 1 with sets C_1, C_2 and Q, which contradicts the definition of similarity for intuitionistic fuzzy sets. The proposed method in this paper produces results consistent with most existing approaches. Moreover, the similarity between the calculated intuitionistic fuzzy set C_1, C_2 and Q is not equal to 1, aligning with the definition of similarity for intuitionistic fuzzy sets and demonstrating good discriminative capability.

Table 3. Comparison of similarity results obtained through different computational methods

	$S_*(C_1,Q)$	$S_*(C_2,Q)$	$S_*(C_3,Q)$	**Classification outcomes**
S_C	0.9500	0.9375	0.9750	P_3
S_{HK}	0.9500	0.9375	0.9500	Indeterminate
S_{LX}	0.9500	0.9375	0.9625	P_3
S_{LO}	0.9293	0.9209	0.9293	Indeterminate
S_{DC}	0.9500	0.9375	0.9750	P_3
S_M	0.9500	0.9375	0.9500	Indeterminate
S_{LS1}	0.9500	0.9375	0.9500	Indeterminate
S_{LS2}	0.9500	0.9375	0.9625	P_3
S_{LS3}	0.9583	0.9542	0.9583	Indeterminate
S_{HY1}	0.9250	0.9250	0	Indeterminate
S_{HY2}	0.8857	0.8857	0.8857	Indeterminate
S_{HY3}	0.8605	0.8605	0.8605	Indeterminate
S_Y	0.9906	0.9871	0.9959	P_3
S_{ZY}	N/A	N/A	N/A	Indeterminate
S_{CR}	0.9469	0.9350	0.9608	P_3
S_{BA}	0.9500	0.9375	0.9667	P_3
$WSN_1^{[15]}$	0.7295	0.7295	0.8143	P_3
$WSN_2^{[15]}$	0.7295	0.7295	0.8143	P_3
$S_{LM}^{[17]}$	0.9000	0.8906	0.9063	P_3
$S_L^{[11]}$	0.9500	0.9375	0.9625	P_3
S_{SPP}	0.6350	0.5525	0.7263	P_3

Example 4-3: The numbers C_1, C_2, and C_3 represent three types of intuitive fuzzy patterns $P_1, P_2 a$ and P_3 defined over the domain $X=\{x_1,x_2,x_3\}$. Q denotes an unknown pattern classified as one of the modes C_1, C_2 or C_3, as illustrated below.

$$C_1=\{\langle x_1,0.5,0.3\rangle,\langle x_2,0.7,0\rangle,\langle x_3,0.4,0.5\rangle,\langle x_4,0.7,0.3\rangle\}$$
$$C_2=\{\langle x_1,0.5,0.2\rangle,\langle x_2,0.6,0.1\rangle,\langle x_3,0.2,0.7\rangle,\langle x_4,0.7,0.3\rangle\}$$
$$C_3=\{\langle x_1,0.5,0.4\rangle,\langle x_2,0.7,0.1\rangle,\langle x_3,0.4,0.6\rangle,\langle x_4,0.7,0.2\rangle\}$$
$$Q=\{\langle x_1,0.4,0.3\rangle,\langle x_2,0.7,0.1\rangle,\langle x_3,0.3,0.6\rangle,\langle x_4,0.7,0.3\rangle\}$$

Comparison of similarity results obtained through different calculation methods, as presented in Table 3.

The similarity calculation results are presented in Table 3. The similarity rankings for items $S_{SPP}(C_1,Q)=0.6350$, $S_{SPP}(C_2,Q)=0.5525$, and $S_{SPP}(C_3,Q)=0.7263$ are as follows: $S_{SPP}(C_3,Q)>S_{SPP}(C_1,Q)>S_{SPP}(C_2,Q)$ indicating that item P_3 is most similar to item Q.

Analysis: This dataset is an extreme case, where the membership or non-membership degrees of certain intuitive fuzzy numbers are equal. As shown in Table 3, the values for S_{HK}, S_{LO}, S_M, S_{LS1}, S_{LS3}, S_{HY1}, S_{HY2}, S_{HY3} and S_{ZY} are indeterminate. Although the similarity measure WSN_1 between WSN_2 and P_3 is closest to Q, it does not allow for further discrimination of the similarity relationships among P_1, P_2 and Q. The proposed method yields results consistent with most existing approaches and demonstrates higher accuracy, enabling a more definitive assessment of the similarity relationships among P_1, P_2 and Q.

Example 4-4: The numbers C_1, C_2, and C_3 represent three types of intuitive fuzzy patterns $P_1, P_2 a$ and P_3 defined over the domain $X=\{x_1,x_2,x_3\}$. Q denotes an unknown pattern classified as one of the modes C_1, C_2 or C_3, as illustrated below.

$$C_1=\{\langle x_1,0.34,0.34\rangle,\langle x_2,0.19,0.48\rangle,\langle x_3,0.02,0.12\rangle\}$$
$$C_2=\{\langle x_1,0.35,0.33\rangle,\langle x_2,0.20,0.47\rangle,\langle x_3,0,0.14\rangle\}$$
$$C_3=\{\langle x_1,0.33,0.35\rangle,\langle x_2,0.21,0.46\rangle,\langle x_3,0.01,0.13\rangle\}$$
$$Q=\{\langle x_1,0.37,0.31\rangle,\langle x_2,0.23,0.44\rangle,\langle x_3,0.04,0.10\rangle\}$$

Comparison of similarity results obtained through different calculation methods, as presented in Table 4.

The similarity calculation results are presented in Table 4. The similarity rankings for items $S_{SPP}(C_1,Q)=0.9418$, $S_{SPP}(C_2,Q)=0.9388$, and $S_{SPP}(C_3,Q)=0.9412$ are as follows: $S_{SPP}(C_1,Q)>S_{SPP}(C_3,Q)>S_{SPP}(C_2,Q)$ indicating that item P_1 is most similar to item Q.

Analysis: This dataset is an extreme case, with the difference between membership and non-membership degrees in the intuitionistic fuzzy numbers being approximately 0.01, necessitating high computational precision. As shown in Table 4, cases S_C, S_{HK}, S_{LX}, S_{LO}, S_{DC}, S_M, S_{LS1}, S_{LS2}, S_{LS3}, S_{HY1}, S_{HY2}, S_{HY3}, S_{CR}, S_{BA}, WSN_1, WSN_2, S_{LM} and S_L are indeterminate. The proposed method maintains good discriminative ability even for data with high similarity, and the results are consistent with those obtained by most existing approaches.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202510_11(10).0011

Table 4. Comparison of similarity results obtained through different computational methods

	$S_*(C_1,Q)$	$S_*(C_2,Q)$	$S_*(C_3,Q)$	**Classification outcomes**
S_C	0.9700	0.9700	0.9700	Indeterminate
S_{HK}	0.9700	0.9700	0.9700	Indeterminate
S_{LX}	0.9700	0.9700	0.9700	Indeterminate
S_{LO}	0.9700	0.9700	0.9700	Indeterminate
S_{DC}	0.9700	0.9700	0.9700	Indeterminate
S_M	0.9700	0.9700	0.9700	Indeterminate
S_{LS1}	0.9700	0.9700	0.9700	Indeterminate
S_{LS2}	0.9700	0.9700	0.9700	Indeterminate
S_{LS3}	0.9800	0.9800	0.9800	Indeterminate
S_{HY1}	0.9700	0.9700	0.9700	Indeterminate
S_{HY2}	0.9533	0.9533	0.9533	Indeterminate
S_{HY3}	0.9418	0.9418	0.9418	Indeterminate
S_Y	0.9892	0.9745	0.9819	P_1
S_{ZY}	0.9414	0.9410	0.9412	P_1
S_{CR}	0.9699	0.9699	0.9699	Indeterminate
S_{BA}	0.9700	0.9700	0.9700	Indeterminate
WSN_1[15]	0.9103	0.9103	0.9103	Indeterminate
WSN_2[15]	0.9103	0.9103	0.9103	Indeterminate
S_{LM}[17]	0.8441	0.8441	0.8441	Indeterminate
S_L[11]	0.9700	0.9700	0.9700	Indeterminate
S_{SPP}	0.9418	0.9388	0.9412	P_1

5. Multi-attribute Decision-making Method based on SPP-IFS

This paper proposes that the SPP-IFS similarity measurement method exhibits high discriminative power and computational accuracy. Subsequently, it considers the integration of the SPP-IFS with a pre-evaluation approach based on fused subtraction set potentials and the TOPSIS method. A novel multi-attribute decision-making approach based on intuitionistic fuzzy sets is introduced. This method is applied to bridge risk assessment to determine the priority of maintenance, offering new perspectives and strategies for decision-makers addressing complex multi-attribute decision problems.

Suppose there are m alternative solution sets denoted as $A_i=\{A_1,\ldots,A_m\}, i=1,2,\ldots,m$, with evaluation criteria set $G_j=\{G_1,G_2,\ldots,G_n\}, j=1,2,\ldots,n$, where each criterion is assigned a weight represented by $w=(w_1,w_2,\ldots,w_n)$, and with $\sum_{j=1}^{n} w_j=1$.

Step one: construct the evaluation information matrix $D=(x_{ij})_{m\times n}$ of the intuitionistic fuzzy set, where x_{ij} denotes the intuitionistic fuzzy number of the i-th alternative relative to the j-th criterion.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202510_11(10).0011

Step Two: Determine the ideal and negative ideal solutions of the intuitionistic fuzzy set. The calculation formulas for the ideal and negative ideal solutions are as follows:

$$Z^{+}=\left\{\left\langle \max_{i} u_{ij}, \min_{i} v_{ij} \right\rangle \mid j=1,2,\ldots,n\right\} \tag{6}$$

$$Z^{-}=\left\{\left\langle \min_{i} u_{ij}, \max_{i} v_{ij} \right\rangle \mid j=1,2,\ldots,n\right\} \tag{7}$$

Step three: Calculate weights using the entropy weight method. This approach determines indicator weights based on the degree of information variation among the indicators, thereby minimizing the influence of subjective human factors on the evaluation results.

The intuitive fuzzy entropy formula for the evaluation values of each attribute is:

$$E(v_{ij})=\frac{1-|\mu_{ij}-v_{ij}|^{2}+\pi_{ij}^{2}}{2} \tag{8}$$

The average intuitive fuzzy entropy for each scheme concerning individual attributes is:

$$E(r_j)=\frac{1}{m}\sum_{i=1}^{m}E(v_{ij}) \tag{9}$$

The objective weight calculation formulas for each attribute are as follows:

$$w_j=\frac{1-E(r_j)}{\sum_{j=1}^{n}\left[1-E(r_j)\right]} \tag{10}$$

Step Four: Integrate the subtraction set approach for pre-evaluation of potential levels. Calculate the potential values of the subtraction set pairs between alternative A_i and the positive and negative ideal solutions across various evaluation indices, to determine whether the alternative and the ideal solutions belong to the same potential level. The specific scoring criteria are as follows:

Under evaluation standard G_j, the subtraction set of the intuitive fuzzy numbers corresponding to scheme A_i has a potential value of $s_f(x_{ij})$. The positive ideal solution and negative ideal solution have potential values of $s_f^{+}(x_j)$ and $s_f^{-}(x_j)$, respectively, for their corresponding subtraction sets under the same evaluation standard G_j. The interval number between potential levels is δ, with $\delta=0$ for intervals within the same potential level. The scores of the alternative scheme relative to the positive and negative ideal solutions are ψ_{ij}^{+} and ψ_{ij}^{-}.The specific pre-scoring methodology is as follows:

(1)The preliminary scores of $s_f(x_{ij})$ and $s_f^{+}(x_j)$ are $\psi_{ij}^{+}=1-\delta/4$.

(2)The preliminary scores of $s_f(x_{ij})$ and $s_f^-(x_j)$ are $\psi_{ij}^- = \delta/4$.

(3)The comprehensive preliminary score for A_i is rated as $\psi_i = \frac{\sum_{j=1}^{n}\left(\psi_{ij}^+ + \psi_{ij}^-\right)}{n}$.

Step five: Utilize the proposed formula (5) to calculate the similarity $S_{SPP}(A_i, Z^+)$ between each alternative solution and the positive ideal solution, as well as the similarity $S_{SPP}(A_i, Z^-)$ between each alternative and the negative ideal solution.

Step six: Calculate the adhesion progress using the following formula:

$$C\left(A_i\right) = \frac{D^-{}_{SPP}\left(A_i\right)}{D^+{}_{SPP}\left(A_i\right) + D^-{}_{SPP}\left(A_i\right)} \tag{11}$$

Where: $D^+{}_{SPP}(A_i) = 1 - S_{SPP}(Z^+, A_i)$, $D^-{}_{SPP}(A_i) = 1 - S_{SPP}(A_i, Z^-)$.

Step Seven: The final score for each alternative is calculated and then sorted by the following formula:

$$Y_i = \omega' C\left(A_i\right) + \omega'' \psi_i \tag{12}$$

In the formula, ω' and ω'' represent the predetermined weights assigned to the final score, while a larger Y_i indicates a higher level of risk.

Taking the issue of bridge risk assessment as an example, to ensure public safety, it is essential to conduct regular evaluations of bridge risks. This approach guarantees that bridges identified as high-risk receive timely maintenance. Determining the priority sequence for structural repairs is crucial. A bridge management authority responsible for maintenance and repairs must assess and evaluate seven bridges within its jurisdiction—namely, bridges $A_1, A_2, \ldots, A_7$. Based on experimental data and literature references [22-23], four key indicators are selected: G_1 (safety), G_2 (functionality), G_3 (sustainability), and G_4 (environmental factor)s.

The comprehensive evaluation information matrix of the seven bridges under four evaluation indicators, represented as an intuitionistic fuzzy set, is shown in Table (6).

Table 5. Comprehensive Evaluation Matrix of Intuitionistic Fuzzy Sets

	G_1	G_2	G_3	G_4
A_1	$\langle 0.469, 0.316 \rangle$	$\langle 0.536, 0.249 \rangle$	$\langle 0.469, 0.431 \rangle$	$\langle 0.670, 0.204 \rangle$
A_2	$\langle 0.379, 0.350 \rangle$	$\langle 0.335, 0.393 \rangle$	$\langle 0.406, 0.323 \rangle$	$\langle 0.305, 0.317 \rangle$
A_3	$\langle 0.430, 0.293 \rangle$	$\langle 0.406, 0.413 \rangle$	$\langle 0.476, 0.262 \rangle$	$\langle 0.415, 0.385 \rangle$
A_4	$\langle 0.536, 0.282 \rangle$	$\langle 0.507, 0.312 \rangle$	$\langle 0.622, 0.149 \rangle$	$\langle 0.346, 0.263 \rangle$
A_5	$\langle 0.500, 0.300 \rangle$	$\langle 0.600, 0.192 \rangle$	$\langle 0.435, 0.196 \rangle$	$\langle 0.700, 0.100 \rangle$
A_6	$\langle 0.200, 0.483 \rangle$	$\langle 0.400, 0.329 \rangle$	$\langle 0.500, 0.341 \rangle$	$\langle 0.273, 0.427 \rangle$
A_7	$\langle 0.435, 0.465 \rangle$	$\langle 0.500, 0.500 \rangle$	$\langle 0.400, 0.270 \rangle$	$\langle 0.500, 0.189 \rangle$

The feasible ideal and negative ideal solutions calculated according to formulas (6) and (7) are:
Calculate the weights of each indicator, denoted as w, using the entropy weight method formula (8-10).

$$w = \langle 0.2389, 0.2413, 0.2330, 0.2868 \rangle$$

Preliminary scoring of each bridge design is conducted based on the fusion subtraction set approach for potential evaluation. The similarity between each bridge scheme and the positive ideal solution $S_{SPP}^{+}(A_i)$, as well as the similarity to the negative ideal solution $S_{SPP}^{-}(A_i)$, is calculated using Equation (5). The relative closeness of each scheme is then determined via Equation (11), with results rounded to four decimal places, as follows:

Table 6. Distances between each bridge design solution and the positive and negative ideal solutions

	ψ_i	$S_{SPP}^{+}(A_i)$	$S_{SPP}^{-}(A_i)$	$C(A_i)$
A_1	0.2	0.8287	0.7757	0.5670
A_2	0.15	0.7548	0.9007	0.2883
A_3	0.175	0.7847	0.8583	0.3969
A_4	0.225	0.8305	0.7756	0.5697
A_5	0.25	0.8712	0.7017	0.6985
A_6	0.1125	0.7081	0.9169	0.2217
A_7	0.175	0.7920	0.8428	0.4305

Calculate the final scores of various bridge risk mitigation strategies using Equation (12), setting $\omega' = 0.8$ and $\omega'' = 0.2$.

Table 7. Final Risk Assessment Scores for Bridges

	A_1	A_2	A_3	A_4	A_5	A_6	A_7
Y_i	0.6536	0.3806	0.4925	0.6807	0.8088	0.2898	0.5194

The final risk assessment scores for bridges, as shown in Table 7, indicate the risk ranking of various bridges.

$$A_5 > A_4 > A_1 > A_7 > A_3 > A_2 > A_6$$

The lower the final rating of a bridge structure, the better; therefore, maintenance efforts should prioritize bridges with higher final scores. The priority ranking for the seven bridges is: $A_5 > A_4 > A_1 > A_7 > A_3 > A_2 > A_6$. Based on the final ratings, Bridge A_5 has the highest score and should be prioritized for maintenance, while Bridge A_6 has the lowest score, indicating it is considered last in maintenance priority. This outcome aligns with the findings in reference [23], which classifies the risk of the seven bridges into only three categories. Consequently, the proposed method demonstrates certain advantages over the approach in reference [23], which is limited by its three-risk-level classification system.

6. Conclusion

Traditional similarity measurement methods for intuitionistic fuzzy sets often encounter challenges such as difficulty in distinguishing samples and insufficient computational accuracy when dealing with certain specialized data types. To address these issues, it is necessary to introduce new parameters for quantifying intuitionistic fuzzy sets. These sets express fuzziness through membership degree, non-membership degree, and hesitation degree, with the relative certainty states and developmental trends also serving as influential factors in the similarity assessment between intuitionistic fuzzy sets. Consequently, the SPP-IFS similarity calculation method demonstrates sound theoretical justification. To solve multi-attribute decision-making problems, this paper proposes a TOPSIS decision model that integrates the difference set potential pre-scoring method with the SPP-IFS similarity calculation approach, offering a novel perspective and approach for multi-attribute decision analysis. Currently, this method is applied solely to the similarity computation of intuitionistic fuzzy sets; future research may extend its application to interval-valued intuitionistic fuzzy sets, interval hesitant intuitionistic fuzzy sets, and interval trapezoidal intuitionistic fuzzy sets, as well as incorporate it with methods such as VIKOR and three-criteria decision models.

Acknowledgments

The asterisk (*) indicates corresponding author. This study was supported by funding from the Hebei Province Key Laboratory of Data Science and Applications and the Tangshan City Key Laboratory of Data Science, under project numbers 10120201 and 10120301.

References

[1] Zadeh L A. Fuzzy Sets[J]. Information and control, 1965, 8(3): 338-353.

[2] Atanassov K T. Intuitionistic fuzzy sets[M].Physica-Verlag HD, 1999.

[3] Atanassov K T. More on intuitionistic fuzzy sets[J]. Fuzzy sets and systems, 1989, 33(1): 37-45.

[4] Wang Zhiping,Fu Min,Wang Peiwen.Multi-attribute Group Decision Making Model Based on Prospect Theory and TOPSIS in Probabilistic Hesitating Fuzzy Environmen[J].Science Technology and Engineering.2022,(4):1329-1337.

[5] Li Baoping,Chen Huayou.Intuitionistic Fuzzy Area Set and Its Application in Pattern Recognition[J].Fuzzy Systems and Mathematics,2015,29(01):134-141.

[6] Jiang Xiaowei,Chen Zichun,Li Linyu.Intuitionistic Fuzzy Value Similarity Based on Einstein Operator and Its Application in Clustering[J].Journal of Sichuan Normal University (Natural Science).2024,(4):528-536.

[7] Chen S M, Tan J M. Handling multicriteria fuzzy decision-making problems based on vague set theory[J]. Fuzzy sets and systems, 1994, 67(2): 163-172.

[8] Chen S M. Measures of similarity between vague sets[J]. Fuzzy sets and Systems, 1995, 74(2): 217-223.

[9] Bustince H, Burillo P. Vague sets are intuitionistic fuzzy set[J]. Fuzzy Set and Systems, 1996, 79(3):403-405.

[10] Hong D H , Kim C .A Note on Similarity Measures Between Vague Sets and between Elements[J]. Information Sciences, 1999, 115(1-4):83-96.

[11] Li Fan,Xu Zhangyan.Measures of Similarity between Vague Sets[J].Journal of Software,2001,(06):922-927.

[12] Chen S M, Randyanto Y. A novel similarity measure between intuitionistic fuzzy sets and its applications[J]. International Journal of Pattern Recognition and Artificial Intelligence, 2013, 27(07): 1350021.

ISSN: 2414-1895 DOI: 10.6919/ICJE.202510_11(10).0011

[13] Hwang C M, Yang M S, Hung W L, et al. A Similarity Measure of Intuitionistic Fuzzy Sets Based on the Sugeno Integral with Its Application to Pattern Recognition[J]. Information Sciences, 2012, 189: 93-109.

[14] Hung W L , Yang M S .Similarity Measures of Intuitionistic Fuzzy Sets Based on Hausdorff Distance[J]. Pattern Recognition Letters, 2004, 25(14):1603-1611.

[15] Yang Yong,Zhang Dongsheng,Wang Yanru,et al.Similarity measure of intuitionistic fuzzy sets based on sine function and its application[J].Computer Engineering and Science.2018,(1):133-138.

[16] Chen, Shyi-Ming, and Chia-Hao Chang. "A novel similarity measure between Atanassov's intuitionistic fuzzy sets based on transformation techniques with applications to pattern recognition." Information Sciences 291 (2015): 96-114.

[17] Liang Meishe,Mi Jusheng,Jin Chenxia.A Novel Similarity Measure between Intuitionistic Fuzzy Sets with Application to Pattern Recognition[J].Fuzzy Systems and Mathematics.2020,(6):157-166.

[18] Zhao Keqin.Set Pair Analysis and Its Preliminary Application[J].Exploration of Nature,1994,(01):67-72.

[19] Jin Juliang,Guo Han,Li Zheng,et al.Evaluating the Dynamic Carrying Capacity of Water Resources Using the Five-variable Gravitation Subtraction Set Pair Potential Method[J].Journal of Irrigation and Drainage.2021,(6):1-7.

[20] Liu Xiumei.Multi-Attribute Decision Making Method Based on Intuitionistic Fuzzy Numbers and Connection Numbers[J].Journal of Lianyungang Teachers College.2008,(4):91-94.

[21] Chen S M, Chang C H. A novel similarity measure between Atanassov's intuitionistic fuzzy sets based on transformation techniques with applications to pattern recognition[J]. Information Sciences, 2015, 291: 96-114.

[22] Wen Yaqing.Multi-criteria decision making method based on intuitionistic fuzzy sets and its application[D].Journal of North Minzu University,2024.

[23] Shen F, Xu J, Xu Z. An outranking sorting method for multi-criteria group decision making using intuitionistic fuzzy sets[J]. Information Sciences, 2016, 334: 338-353.