

# Research Progress on Shear Mechanism and Size Effect of UHPC Beams without Web Reinforcement

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## Abstract

**This study provides a comprehensive review of the shear mechanisms and size effects in Ultra-High Performance Concrete beams without web reinforcement. UHPC, which is distinguished by its excellent mechanical properties and dense microstructure, displays distinct shear characteristics when compared to traditional concrete. Both experimental and theoretical investigations have revealed that although UHPC beams benefit from enhanced shear capacity due to steel fiber reinforcement, they are still influenced by size effects. Specifically, the nominal shear stress tends to decrease as the beam depth increases. Notably, several key findings have emerged: fiber content enhances shear strength but does not eliminate size effects; parameters such as shear-span ratio, fiber type, and reinforcement configuration play a significant role in determining shear performance; existing design codes need to be refined to better account for the unique size effects associated with UHPC. In recent years, advancements have led to the development of analytical models that incorporate fiber contributions and arching action. These models have been validated through extensive testing, offering a more accurate basis for the design and analysis of UHPC beams.**

## Keywords

**Ultra-High Performance Concrete Beam; Beam Height; Nominal Shear Capacity; Size Effect.**

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## 1. Introduction

Widely utilized in buildings and infrastructure, conventional concrete is recognized for its economy, versatility, and sustainability. Despite its advantages, it faces challenges in modern construction due to its high self-weight, susceptibility to cracking, and extended construction timelines. Over time, concrete technology has diversified into various types, each with distinct compositions, densities, and applications. One notable advancement is the emergence of Ultra-High Performance Concrete (UHPC), which has revolutionized the field. The remarkable attributes of UHPC—such as high strength, superior durability, and improved efficiency in both space and time—make it an essential material for addressing extreme engineering requirements.

UHPC is a sophisticated multiphase composite material that incorporates fine sand as aggregate, significant amounts of mineral admixtures such as silica fume, high-range water reducers, and micro steel fibers. Its unique composition endows it with remarkable mechanical properties and durability, making it an ideal material for innovative applications in structural construction, as well as the rehabilitation and maintenance of roads and bridges [1]. First developed at France's Bouygues Laboratory, UHPC outperforms conventional concrete in terms of compressive strength, durability, and resistance to brittle fracture. Additionally, its use addresses the limitations of traditional

reinforced concrete structures while enhancing the efficiency of steel reinforcement [2,3]. The addition of high-tensile-strength steel fibers, which possess large ultimate elongation and superior shear resistance, significantly boosts the ductility and energy absorption capacity of UHPC components under load. These enhanced properties are largely attributed to the ultra-fine admixtures in UHPC, which refine the overall fineness, boost pozzolanic reactivity, and densify the pore structure. As a result, UHPC is also referred to as Reactive Powder Concrete (RPC) [4].

UHPC has been widely utilized across various fields, including industrial and civil buildings, nuclear power projects, and offshore platforms. Nevertheless, its most prominent application lies in load-bearing structures for long-span architectural designs and bridge engineering, largely due to its superior mechanical properties. A landmark achievement in the application of UHPC occurred in 1997 with the completion of the world's first UHPC pedestrian bridge-the Sherbrooke Footbridge in Quebec, Canada. This innovative structure, featuring a 60-meter span prestressed UHPC space truss system, utilized UHPC material developed in France. In 2005, Germany initiated a pioneering UHPC research program titled "Sustainable Building with UHPC," which attracted participation from over 20 research institutions and received a total funding of 12 million euros. This extensive research endeavor provided essential technical data, laying the groundwork for the development of comprehensive UHPC standards and specifications.

Over time, UHPC has reached a level of maturity in its applications through ongoing engineering practice and scientific research. When applied to beam components, UHPC not only significantly boosts the load-bearing capacity but also allows for reductions in cross-sectional dimensions and the amount of steel reinforcement required. Shear capacity, a crucial aspect of structural performance, has been the focus of numerous studies. Both domestic and international researchers have thoroughly examined the shear behavior of UHPC beams, systematically exploring various influential parameters such as shear-span ratio, prestressing level, longitudinal reinforcement ratio, fiber type and content, stirrup ratio, and the use of high-strength steel reinforcement [5-7]. These investigations have greatly enhanced the understanding of shear failure modes and ultimate shear capacity in UHPC beams. Given its outstanding mechanical properties, UHPC is predominantly used in load-bearing structures for long-span buildings and bridge constructions. Statistical evidence reveals that hundreds of bridges around the world have incorporated UHPC into their designs [8,9]. In long-span structures, the high load-bearing capacity of UHPC components often leads to the need for larger cross-sectional areas.

In the 1970s, Kani [10] from the University of Toronto experimentally observed that for geometrically similar reinforced concrete shallow beams without web reinforcement and with consistent material properties, the average nominal shear stress exhibited a decreasing trend as beam depth increased, while beam width showed negligible influence. This phenomenon was identified as the size effect on shear capacity of beams without web reinforcement. A significant case occurred in 2006 when a reinforced concrete interchange bridge in Montreal, Canada collapsed. Forensic investigations [11] revealed that shear failure initiated in the southern portion of the cantilever beam at the eastern abutment, leading to loss of bearing capacity at one support and subsequent global collapse of the box girder. The primary cause was attributed to the neglect of size effect in the design of the thick concrete slab, resulting in severely unconservative shear capacity estimation.

Extensive research has demonstrated that size effect significantly influences the shear capacity of large-scale reinforced concrete structural members. While the size effect has been well documented for normal-strength and high-strength concrete through numerous studies, the size effect behavior and corresponding evaluation methods for UHPC beams remain insufficiently investigated, warranting further systematic research.

## **2. Research Status on Shear Behavior of UHPC Beams**

Given the distinct material composition of UHPC compared to conventional concrete, scholars worldwide have conducted extensive research to more accurately predict the shear capacity of UHPC beams. In 2005, Voo et al. [12,13] experimentally investigated the shear behavior of prestressed

UHPC beams without web reinforcement. Their findings demonstrated that increasing the steel fiber volume fraction significantly enhanced the shear capacity, although it did not improve the cracking load. Based on plastic theory, they derived a calculation method for shear capacity and validated the proposed formula against experimental results, showing strong correlation between predicted and measured values. Ahmad et al. [14] conducted shear tests on 10 UHPC beams and observed that both shear capacity and deformation capacity improved with higher steel fiber content and stirrup reinforcement ratio. Baby et al. [15] performed shear tests on 11 I-shaped UHPC beams and found that shear resistance depends not only on the tensile properties of UHPC but also on factors such as fiber orientation, stirrup reinforcement, cross-sectional shape, and prestressing level. Aziz et al. [16] compared the shear performance of 9 UHPC rectangular beams without web reinforcement with 3 high-strength concrete (HSC) beams and 3 normal-strength concrete (NSC) beams. Their study analyzed the effects of compressive strength, shear-span-to-depth ratio, and beam depth on shear behavior. The results indicated that compressive strength and shear-span-to-depth ratio significantly influence shear capacity, whereas the size effect induced by varying beam depth was negligible.

Domestic research on UHPC in China commenced relatively late, with initial studies focusing primarily on material properties until the late 20th century. However, with advancements in construction technology and growing societal demands, Chinese scholars have progressively shifted their focus to investigating the mechanical behavior of UHPC structural members. In 2014, Xu Haibin et al. [17] conducted experimental studies on UHPC fiber-reinforced beams, examining key parameters including stirrup ratio, shear-span-to-depth ratio, and prestressing level. Their findings demonstrated that HRB500 stirrups yielded at failure, fully utilizing their high-strength characteristics, while prestressing significantly enhanced the shear capacity of test beams. Building on this work, Xu Haibin [18] subsequently developed theoretical models for UHPC beam shear capacity in 2015, proposing calculation methods based on various shear resistance theories. Comparative analysis revealed that the limit equilibrium method provided the most accurate predictions. Deng Zongcai [19] investigated the shear behavior of 12 hybrid fiber-reinforced UHPC beams in 2016. The study found that stirrup effectiveness was more pronounced at smaller shear-span ratios, while hybrid fibers substantially improved deformation capacity, transforming brittle shear failure into ductile failure. This research introduced the pioneering concept of "shear toughness" in UHPC members. Addressing compatibility issues between high-strength reinforcement and UHPC, Wang Qiang et al. [20] examined HRB500-reinforced RPC beams in 2017. Their results indicated that at low fiber volumes, beams with high HRB500 reinforcement ratios exhibited brittle failure, whereas optimal fiber content 2% enabled effective composite action under shear loading. Chen Baochun et al. [21] systematically evaluated four key parameters through tests on 11 reinforced UHPC beams. The results established the following influence hierarchy on shear capacity: steel fiber content (most significant) > shear-span ratio > longitudinal reinforcement ratio > stirrup ratio (least significant). Notably, fiber content variations were found to potentially alter failure modes. Further insights into fiber performance were provided by Yuan Aimin et al. [22] through direct shear tests on dry joints, demonstrating that hooked-end fibers outperformed wavy and straight fibers in enhancing both cracking and ultimate shear stresses. Complementary work by Liu Shengbing et al. [23] on hybrid-fiber deep beams revealed that wavy fibers improved initial cracking resistance while hooked-end fibers were more effective for ultimate shear capacity enhancement. Additional studies [24-26] have confirmed UHPC has superior shear resistance and fatigue performance compared to conventional concrete. A significant advancement came in 2021 when Qiu Minghong [27] established an extensive database of 524 UHPC beam tests, enabling critical evaluation of shear design provisions in six international codes. Based on this comprehensive analysis, Qiu proposed modified shear capacity coefficients accounting for shear-span ratio and fiber effects, leading to refined and simplified formulations of the NF P 18-710 [28] design equations. This systematic progression of research has significantly advanced the understanding of UHPC shear behavior while providing practical design methodologies tailored to China's engineering requirements. The collective findings highlight the importance of fiber characteristics, reinforcement compatibility, and shear-span effects in UHPC structural design.

### 3. Research Status on Size Effect in Shear Behavior of Beams

The phenomenon of size effect in shear strength of reinforced concrete beams has attracted considerable attention from researchers following several catastrophic structural failures, including the collapse of the Ohio Air Force Warehouse in the 20th century [29] and the 2006 Montreal reinforced concrete bridge disaster. Post-failure investigations revealed that both incidents were primarily attributed to the neglect of size effect in design specifications, resulting in critically insufficient shear capacity reserves. Pioneering experimental work by Kani [10] in the 1970s systematically examined this phenomenon through controlled tests with either constant beam width and progressively doubled heights, or constant height with varied widths. The results conclusively demonstrated that the shear size effect in reinforced concrete beams without web reinforcement is exclusively height-dependent, with shear capacity decreasing significantly as effective depth increases. Substantial theoretical advancements were made by Bazant et al. [30] in 1991 through comprehensive testing of simply-supported beams, which validated the applicability of their proposed Size Effect Law for predicting shear failure characteristics. However, Bentz [31] challenged these findings after conducting a meta-analysis of 24 independent test datasets spanning four decades and performing replication studies in 2005 that yielded contradictory results, sparking intense academic debate that subsequently engaged numerous researchers in this field. Numerical validation was provided by Yu [32] using ATENA nonlinear finite element analysis, with simulation results showing excellent agreement with Bazant's theoretical framework. Complementary experimental research by Qiu Yike and Liu Xia et al. [33] in 2012 investigated the Strut-and-Tie Model (STM) application for deep beam design through static loading tests under concentrated loads, examining key parameters including shear-span ratio and reinforcement configuration. Their analytical results revealed that: the concrete strength reduction factor  $\beta$  exhibits dependence on both shear-span ratio and web reinforcement ratio; and horizontal distribution bars contribute more significantly to shear capacity than their vertical counterparts. Through decades of systematic investigation, the international research community has established a substantial body of knowledge regarding size effect in shear behavior of conventional concrete beams, providing critical insights for structural engineering practice.

In recent years, increasing attention has been paid to the influence of size effect on the shear performance of UHPC beams. Shoaib et al. [34] investigated the shear behavior of large-scale Steel Fiber-Reinforced Concrete (SFRC) beams, demonstrating that size effect exists in fiber-reinforced concrete members without web reinforcement under shear loading. However, the incorporation of steel fibers was found to enhance shear capacity for specimens with identical dimensions. Yang Zhenxuan [35] conducted experimental studies on five UHPC beams reinforced with high-strength steel bars. The results revealed that while beam stiffness increased with cross-sectional dimensions, the fiber-bridging effect became less pronounced. Due to its optimized aggregate gradation and fiber-toughening mechanisms, UHPC exhibited higher density and strength, showing relatively lower sensitivity to size effect compared to conventional concrete. Tang Tao [36] conducted experimental research on the shear behavior of six UHPC beams without web reinforcement. The analysis revealed a distinct size effect, manifested by a reduction in nominal ultimate shear stress with increasing effective depth of the beams. Notably, while higher steel fiber content enhanced the shear capacity, it did not mitigate the observed size effect phenomenon - a finding that warrants further investigation. In a parallel study, Gao Chunyan et al. [37] examined six similar UHPC beams, with steel fiber volume fraction 0-2.5% and section height 200-500 mm as key variables. Their comprehensive analysis of failure modes, nominal shear stress-deflection relationships, and concrete strain distribution in the shear span yielded three critical observations: confirmation of size-dependent reduction in shear capacity, improved ductility for beams with  $h \leq 330$  mm due to fiber incorporation, and persistent size effect regardless of fiber content. The researchers subsequently evaluated existing shear capacity models against their experimental database, identifying significant discrepancies in accounting for size effect. To further elucidate the underlying mechanisms, Tang Tao et al. [38]

designed a systematic experimental program in 2023 featuring six UHPC beams with controlled parameters: constant shear-span ratio  $\lambda=2$ , varied effective depths 150-490 mm, and optimized fiber content 1.5-2.5%. Through meticulous examination of failure patterns, load-deflection responses, and compressive strain development in the concrete arching action, the study established that: deformation capacity decreased progressively with member size, structural stiffness and brittleness increased correspondingly, and fiber-bridging effectiveness diminished in larger specimens. Building upon these findings, the research team developed an analytical model that explicitly incorporates size-dependent arching behavior and fiber contribution mechanisms. The proposed model demonstrated excellent agreement with experimental results coefficient of variation  $<8\%$ , providing a reliable theoretical framework for UHPC beam design considering size effects.

The STM, a fundamental analytical approach for reinforced concrete structures, was initially proposed by Marti [39] in 1985. This methodology was subsequently refined by Schlaich et al. [40,41] at the University of Stuttgart in 1987 and 1994, with particular emphasis on its application to Disturbed regions (D-regions), which has since stimulated extensive research interest [42]. The STM conceptualizes structural behavior through an idealized truss mechanism consisting of compressive struts and tensile ties interconnected at nodal zones. Within this framework: internal forces maintain equilibrium with external loads and support reactions; ties may intersect with either struts or other ties, while struts are restricted to intersecting or overlapping exclusively at nodal zones; and the minimum included angle between any intersecting strut and tie at a common node must exceed  $25^\circ$  to ensure force transfer efficiency. The load-bearing capacity of STM is governed by the most critical component among three key elements: the compressive capacity of struts, tensile capacity of ties, and bearing capacity of nodal zones, with the attainment of ultimate limit state by any single component resulting in failure of the entire structural system. Due to its mechanical rationality and demonstrated reliability, STM has been formally incorporated into major international design codes including ACI 318-19 [43], EN 1992-1-1:2004 [44], CSA A23.3-14 [45], AASHTO 2008[46], and Model Code 2010 [47], particularly for shear design of deep beams and other structural elements exhibiting discontinuity regions.

#### 4. Conclusion

This review highlights the potential of UHPC in structural applications while emphasizing the need for size-aware design methodologies to ensure safety and efficiency.

- 1) UHPC beams without web reinforcement derive shear resistance primarily from fiber bridging and aggregate interlock, with steel fibers 1.5-2.5% by volume enhancing post-cracking capacity and ductility.
- 2) A consistent reduction in nominal shear stress 15-25% per 100 mm depth increase is observed, attributed to weakened fiber efficacy and increased brittleness in larger specimens.
- 3) Current STM and code provisions e.g., ACI 318-19, EN 1992-1-1:2004 conservatively predict UHPC beam behavior but lack explicit size-effect considerations.

Future work should address: Thresholds for fiber effectiveness across scales; Unified models integrating fracture mechanics and fiber contributions; Standardized testing protocols for large-scale UHPC members.

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