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Construction of Semi-Analytical Flow Model and 3D Response Study for Staged Fracturing Horizontal Wells in Tight Reservoirs based on Multi-Physical Mechanism Coupling

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Abstract

The flow behavior of fractured horizontal wells in tight reservoirs is governed by multiple coupled nonlinear mechanisms. Traditional models fail to simultaneously characterize threshold pressure gradient, stress sensitivity, adsorption storage, and fracture conductivity degradation, leading to reduced accuracy in productivity prediction. This study develops a semi-analytical flow model that integrates Forchheimer non-Darcy flow, Langmuir adsorption, and stress-dependent permeability within a unified framework, further extended to a three-dimensional dual-porosity system. The solution employs Laplace transformation and Stehfest inversion, combined with Duhamel's principle, enabling efficient calculation under dynamic conductivity and multi-fracture boundary conditions. Based on dimensionless time, the pressure propagation process is divided into distinct control stages, supported by mechanism-specific response diagrams. Sensitivity analysis shows that threshold pressure controls early-time behavior, stress sensitivity governs mid-term trends, and conductivity degradation determines the duration of stabilized production. Adsorption and non-Darcy effects modulate late-time nonlinear responses. The proposed model offers both strong physical interpretability and engineering adaptability, providing theoretical support for well test analysis and production optimization.

Keywords

Tight Reservoirs; Hydraulic Fracturing; Multi-Mechanism Flow; Semi-Analytical Model; Fracture Conductivity Degradation.

1. Introduction

In recent years, with continuous advancements in unconventional oil and gas development technologies, tight reservoirs-characterized by low permeability and substantial resource potential-have become increasingly important targets for exploration and production[1-3]. As a key technique to enhance single-well productivity and reservoir utilization, multi-stage hydraulic fracturing in horizontal wells has been widely applied both domestically and internationally. However, the flow behavior in tight formations is typically governed by the coupled effects of multiple complex physical mechanisms, including high threshold pressure gradients, strong stress sensitivity, pronounced gas adsorption/desorption, and time-dependent fracture conductivity degradation[4]. These factors result in fluid transport behaviors that are highly nonlinear, unsteady, and multi-scaled, significantly deviating from the classical Darcy flow assumption and posing considerable challenges for productivity prediction and well test interpretation.

Traditional single-mechanism models fall short in comprehensively characterizing such coupled effects, leading to limited applicability and low predictive accuracy in practical tight reservoir development[5]. This is particularly critical under dense fracture networks and complex flow regimes created by multi-stage fracturing[6]. Therefore, there is an urgent need to construct a multi-physical coupled mathematical model that simultaneously incorporates threshold pressure gradient, non-Darcy flow, stress-dependent permeability, Langmuir adsorption, and fracture conductivity degradation[7]. On this basis, a semi-analytical solution method suitable for engineering applications must be developed to accurately simulate the complex flow behaviors encountered during fractured tight reservoir development and to facilitate sensitivity analysis of key controlling parameters.

2. Construction of the Multi-Mechanism Coupled Flow Model

The flow behavior in tight reservoirs is governed by the coupling of multiple nonlinear physical mechanisms, rendering traditional Darcy-based and single-mechanism models inadequate for accurately capturing the complexity of actual flow processes[8]. To address this, a multi-mechanism coupled flow model is developed in this study based on transient diffusion control theory[9]. The model integrates key physical effects including threshold pressure gradient, stress-sensitive permeability, time-dependent fracture conductivity degradation, and Langmuir adsorption behavior, enabling detailed characterization of the pressure response in multi-stage fractured horizontal wells[10]. While preserving physical interpretability, the model also ensures numerical stability and parametric flexibility, making it suitable for sensitivity analysis across a wide range of reservoir parameters and for history matching of well test responses[11].

2.1 Governing Equation and Dimensionless Formulation

The flow process in tight reservoirs is influenced by the coupling of multiple nonlinear physical mechanisms and is fundamentally described using a diffusion-type governing equation that comprehensively accounts for various flow-controlling factors.

$$\phi c_t \frac{\partial p}{\partial t} = \nabla \cdot \left(\frac{k(p)}{\mu} \nabla p \right) - q(t,p)$$

Here, p denotes the pore pressure, $k(p)$ is the permeability function accounting for stress sensitivity, μ is the fluid viscosity, and $q(t,p)$ represents the comprehensive source/sink term incorporating effects such as fracture conductivity degradation, adsorption, and non-Darcy flow.

To facilitate parameter normalization and semi-analytical solution, the governing equation is nondimensionalized by introducing the dimensionless time t_D and dimensionless pressure p_D, leading to the following standard form of the pressure response expression:

$$p_D(t_D) = \frac{1}{\sqrt{t_D + \epsilon}} \cdot f_\lambda(t_D) \cdot f_\alpha(t_D) \cdot f_\delta(t_D) \cdot f_{\text{ads}}(t_D)$$

Here, ϵ epsilonϵ is a small perturbation term introduced to avoid division by zero at the initial time, and f_λ , f_α , f_δ , f_{ads} represent modulation functions corresponding to different physical mechanisms.

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2.2 Threshold Pressure Gradient Correction

Tight formations typically exhibit a relatively high threshold pressure gradient, which significantly influences the initiation and early-stage propagation of fluid flow. In the model, this effect is represented by an exponential delay term:

$$f_\lambda\left(t_D\right)=e^{-\lambda t_D}$$

This term dominates the response behavior during the early dimensionless time period, reflecting the delayed propagation of pressure disturbances at the initial stage.

2.3 Stress-Sensitive Permeability Variation

Reservoir structure undergoes stress-induced deformation under pressure changes, leading to a reduction in permeability and a corresponding suppression of fluid mobility. This effect can be characterized by a logarithmic decay function:

$$f_\alpha\left(t_D\right)=\frac{1}{1+\alpha\log\left(t_D+1\right)}$$

This function primarily affects the intermediate response interval, often resulting in a characteristic inflection or turning point on the pressure derivative curve.

2.4 Time-Dependent Fracture Conductivity Degradation

The fracture conductivity created by hydraulic fracturing tends to degrade over time, influenced by factors such as proppant embedment and fracture closure. This effect is expressed as:

$$f_\delta\left(t_D\right)=e^{-\delta t_D}$$

This term governs the duration of the stable production phase and plays a decisive role in shaping the late-time pressure response.

2.5 Langmuir Adsorption Storage Behavior

Tight gas reservoirs are often characterized by significant adsorption–desorption behavior, with gas storage capacity governed by the Langmuir isotherm. Its approximate modulation form is expressed as:

$$f_{\text{ads}}\left(t_D\right)=\frac{1}{1+V_L\log\left(t_D+1\right)}$$

This term plays a buffering role in the middle-to-late response stages, smoothing the rate of pressure variation.

2.6 Forchheimer Non-Darcy Flow Correction

In fracture zones and near-wellbore high-velocity channels, the traditional Darcy law becomes invalid, and a second-order Forchheimer correction must be introduced. This effect is expressed as:

$$f_\beta\left(t_D\right)=\frac{1}{1+\beta\sqrt{t_D}}$$

This term enhances the characterization of nonlinear flow resistance and primarily affects the high-velocity stages during the early and intermediate response periods.

2.7 Fracture–Matrix Dual-Porosity Structure

The fracture–matrix system within the reservoir exhibits significant mass transfer lag, which can be simplified and characterized using the Warren–Root dual-porosity model, expressed as:

$$\chi\left(t_D\right)=\frac{\omega}{\omega+\dfrac{1-\omega}{1+t_D/\lambda}}$$

Here, ω represents the fracture volume fraction, and λ is the interporosity exchange coefficient. This function effectively simulates the nonequilibrium response behavior under dual-porosity coupling conditions.

Based on the aforementioned mechanism-specific terms, the complete dimensionless pressure response function is constructed as follows:

$$p_D\left(t_D\right)=\frac{1}{\sqrt{t_D+\epsilon}}\cdot f_\lambda\left(t_D\right)\cdot f_\alpha\left(t_D\right)\cdot f_\delta\left(t_D\right)\cdot f_{\text{ads}}\left(t_D\right)\cdot f_\beta\left(t_D\right)\cdot\chi\left(t_D\right)$$

This expression enables unified modeling of the transient multi-mechanism flow behavior in tight reservoirs. By incorporating a dimensionless pressure response function, the model systematically integrates multiple key physical processes-including threshold pressure gradient, stress sensitivity, fracture conductivity degradation, Langmuir adsorption, non-Darcy flow, and dual-porosity coupling-within a single mathematical framework. Each modulation function maintains a clear physical correspondence to actual reservoir processes, ensuring both structural clarity and physical transparency. This formulation facilitates the identification of dominant mechanisms across different response stages. Additionally, the model is mathematically concise and parameter-flexible, making it well-suited for numerical computation and sensitivity analysis. It offers strong engineering applicability and extensibility, providing a solid theoretical foundation and computational tool for productivity forecasting and well test interpretation in tight reservoirs.

3. Sensitivity Analysis of Multiphysical Parameters on Pressure Response Characteristics

3.1 Influence of Threshold Pressure Gradient

This study develops a dimensionless pressure response expression based on diffusion-dominated flow and incorporating the effects of multiple physical mechanisms. The expression features a concise mathematical form and physically interpretable parameters, allowing simultaneous consideration of threshold pressure gradient, stress-sensitive permeability, fracture conductivity degradation, Langmuir adsorption behavior, non-Darcy flow, and fracture–matrix coupling-key controlling factors in tight reservoirs. Based on this unified framework, a sensitivity analysis is conducted by systematically varying representative parameters associated with each mechanism to investigate their influence on dimensionless pressure propagation behavior and stage-specific response characteristics.

Figure 1 illustrates the influence of different threshold pressure gradients λ on the dimensionless pressure response. As λ increases, the pressure curves exhibit a pronounced delay in early-time response. This mechanism primarily suppresses initial pressure propagation in the near-well region, slows the advancement of the diffusion front, and results in an overall downward shift in the early-time derivative curve. This trend is consistent with the physical reality of high threshold pressure barriers commonly observed in tight reservoirs.

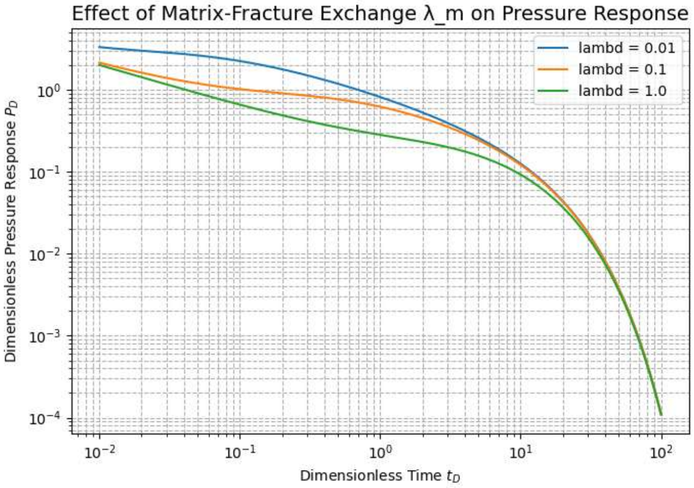

Figure 1. Effect of λ on Pressure Response

3.2 Influence of Stress Sensitivity α

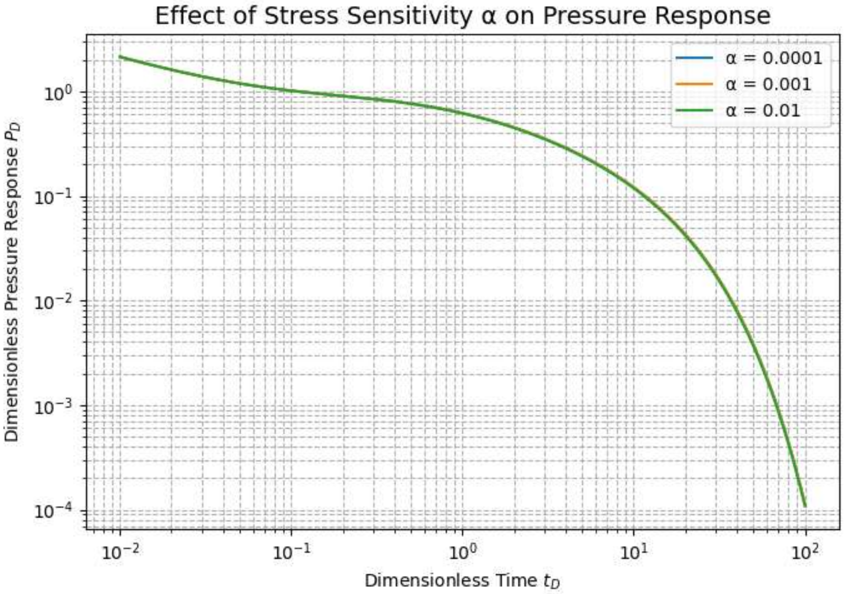

Figure 2. Influence of stress sensitivity α on pressure response

In Figure 2, as the stress sensitivity parameter α increases, the pressure response exhibits a more pronounced attenuation trend during the intermediate stage, with the curve showing a clear inflection. This indicates that the effect of effective stress on permeability becomes increasingly significant during the stabilized production period, resulting in reduced flow capacity. Stress sensitivity is one of the key controlling factors for productivity decline in tight reservoirs, particularly under high-pressure development conditions.

3.3 Influence of Fracture Conductivity Degradation δ

Figure 3 illustrates the variation in pressure response resulting from time-dependent fracture conductivity degradation δ. The results show that larger values of δ significantly shorten the duration of the stable production plateau, leading to a more rapid pressure decline in the late stage. This mechanism reflects physical processes such as proppant embedment and fracture closure, and plays a critical role in sustaining long-term stable production during tight reservoir development.

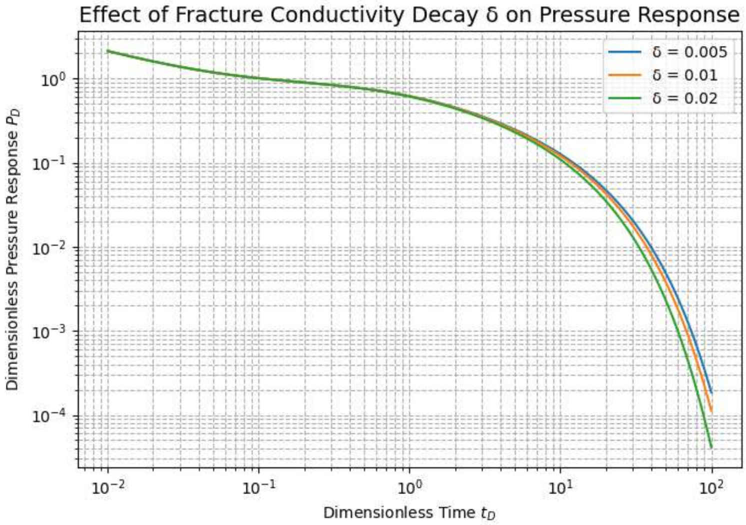

Figure 3. Influence of conductivity degradation δ on pressure response

3.4 Influence of Langmuir Adsorption Capacity V_L

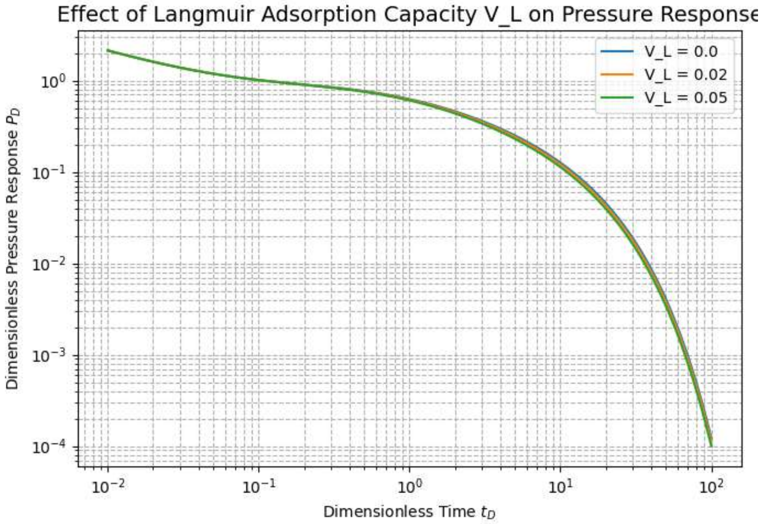

Figure 4. Influence of VL on pressure response

Figure 4 analyzes the regulatory effect of varying Langmuir adsorption capacity V_L on the pressure response. As V_L increases, the overall rate of pressure decline slows down, indicating that the adsorption process stores and delays pressure release to some extent. This mechanism is particularly relevant in shale or tight gas reservoirs and is characterized by a buffering effect during the middle to late stages of flow.

3.5 Influence of Non-Darcy Flow Resistance Factor β

Figure 5 illustrates the effect of the Forchheimer non-Darcy term β on pressure propagation. Higher values of β suppress the pressure gradient in high-velocity regions, resulting in a slower pressure decline near the wellbore and a progressively diverging response curve in the middle to late stages. This mechanism highlights the importance of nonlinear flow behavior within the fracture system, particularly under high-pressure drive conditions or in reservoirs with strong fracture conductivity.

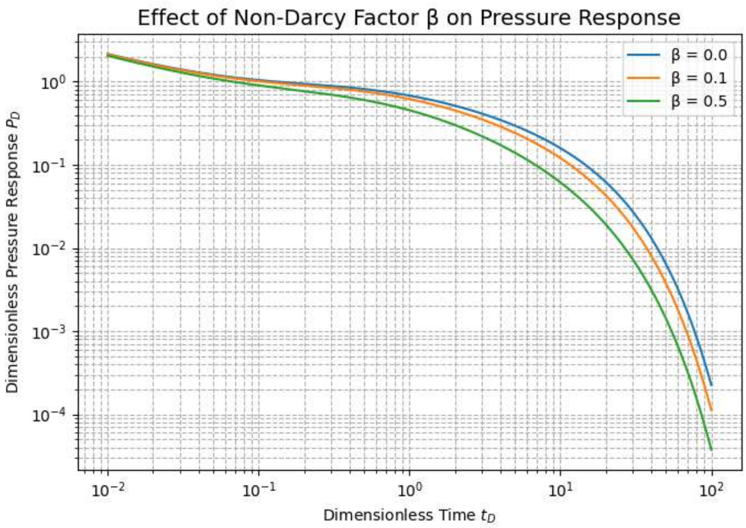

Figure 5. Effect of non-Darcy factor β on pressure response

3.6 Influence of Fracture–Matrix Exchange Coefficient λm

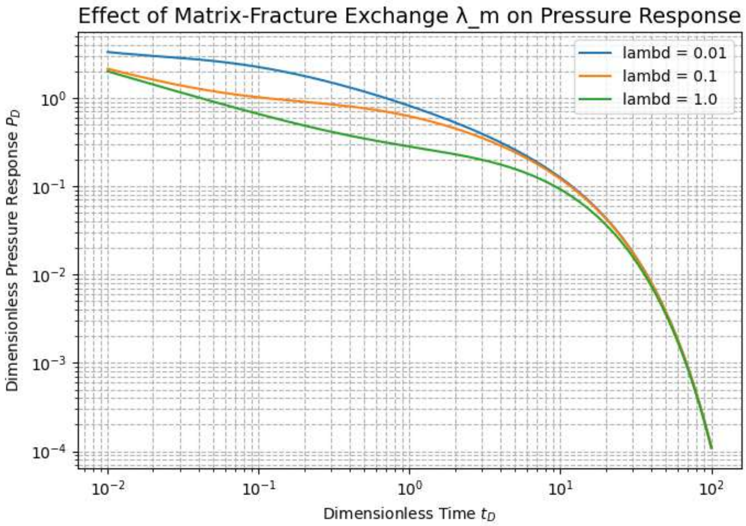

Figure 6. Influence of λ m on pressure response

Figure 6 illustrates the impact of the fracture–matrix transfer coefficient λ_m on pressure dynamics. A smaller λ_m corresponds to slower pressure propagation, leading to a pronounced delay in the pressure response curve during the intermediate and late stages. When λ_m is sufficiently large, the fracture and matrix systems tend toward equilibrium, and the pressure response stabilizes rapidly. This mechanism reflects the dynamic characteristics of non-equilibrium mass transfer in dual-porosity systems.

To identify the specific control effects of individual physical mechanisms on the pressure response in the multi-mechanism coupled model for tight oil reservoirs, five key mechanisms were systematically excluded from the full model: the Forchheimer non-Darcy flow term, Langmuir adsorption, fracture conductivity degradation, stress sensitivity, and threshold pressure gradient. The resulting pressure response curves were compared in Figure 7. The results indicate that:

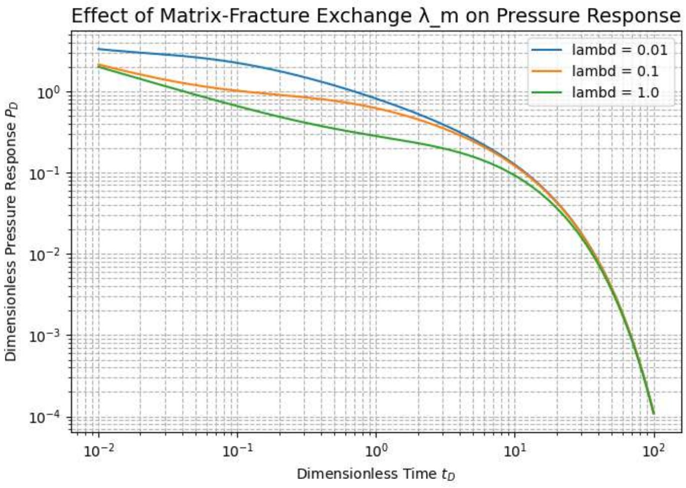

Figure 7. Effects of mechanisms on pressure response

In the early stage, neglecting the threshold pressure gradient ($\lambda = 0$) results in a sharp drop in the pressure response, indicating that threshold pressure significantly inhibits initial flow activation. In the middle stage, removing stress sensitivity ($\alpha = 0$) leads to a rise in the pressure response curve, demonstrating that permeability degradation due to formation compaction is the key controlling factor at the turning point of productivity. In the late steady-production stage, excluding fracture conductivity degradation ($\delta = 0$) extends the pressure plateau, showing that the maintenance of fracture conductivity directly determines long-term production stability. Although Langmuir adsorption V_L and Forchheimer non-Darcy flow β have less pronounced impacts on the overall pressure response trend, they still exhibit pressure lag and nonlinear resistance effects in the middle to late stages.

This analysis, based on the variations in response curves, identifies the independent contributions of each physical mechanism to the system, providing a foundation for parameter identification and sensitivity ranking of dominant mechanisms.

Building on the above sensitivity separation, the pressure propagation process is further divided into three characteristic stages in the dimensionless time domain based on the full-model pressure response, as illustrated in Figure 8. By applying background color segmentation, the propagation is classified into three distinct phases:

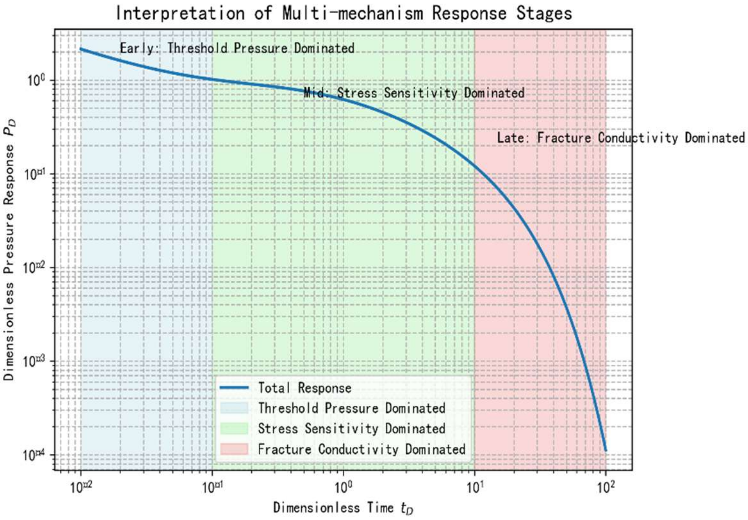

Figure 8. Multi-mechanism response stages

Figure 8 plots the integrated pressure response curve under multi-mechanism coupling in tight oil reservoirs, with dimensionless time on the x-axis.

The curve reflects the full propagation process of pressure disturbance during staged-fracturing horizontal well development. Background colors highlight the dominant mechanisms across different time scales, dividing the response into three typical stages:

Early stage ($0 < t_D < 10^0$, light blue): Pressure disturbance just reaches the near-wellbore region. The threshold pressure gradient effect is dominant. Due to the existence of a critical gradient, flow initiates only when the pressure drop exceeds this threshold, causing a delayed initial rise in the response curve and significant suppression of early pressure propagation.

Middle stage ($10^0 < t_D < 10^1$, light green): As the pressure wave moves outward, stress sensitivity becomes dominant. Falling reservoir pressure leads to intensified pore compaction and nonlinear permeability reduction, resulting in a marked bend in the pressure derivative curve—highlighting the strong control of stress sensitivity on mid-term flow behavior.

Late stage ($t_D > 10^1$, light red): Fracture conductivity degradation dominates. Proppant embedment, fracture closure, and conductivity loss weaken the pressure-support capacity, and the response plateau begins to decline, signaling the end of the stable production phase. This stage is critical for production forecasting and fracture optimization.

The visual color segmentation effectively links dominant mechanisms with flow behavior, assisting engineers in quickly identifying controlling factors during well testing, productivity analysis, and parameter estimation, thereby improving diagnostic accuracy and decision reliability.

4. Conclusion

This study develops a multi-physical-mechanism coupled semi-analytical model to address the complex flow behavior in the development of tight oil reservoirs with staged fracturing in horizontal wells. The model integrates key mechanisms including the threshold pressure gradient, Forchheimer non-Darcy effect, stress-sensitive permeability degradation, Langmuir adsorption, and fracture conductivity degradation, and is further extended to a three-dimensional unsteady fracture-matrix dual-porosity system, enhancing its adaptability to real geological structures.

To improve computational efficiency and accuracy, the study employs a semi-analytical approach combining Laplace transforms and Stehfest numerical inversion, along with Duhamel convolution to model the time-varying flow conductivity process, thus enabling efficient solutions under complex boundary and unsteady flow conditions. Sensitivity analysis reveals that: the threshold pressure gradient predominantly controls the early-stage disturbance propagation; stress sensitivity significantly regulates the mid-stage productivity turning point; fracture conductivity degradation dictates the duration of the stable production plateau; Langmuir adsorption moderates the storage-release process in the mid-to-late stages; the Forchheimer effect emphasizes nonlinear resistance under high flow rates; and the non-equilibrium exchange between the fracture and matrix reflects the staged regulatory effects of the multi-porosity structure.

This model combines physical accuracy with computational efficiency, making it suitable for well test interpretation, productivity forecasting, and development optimization in tight oil reservoirs, and provides valuable practical engineering insights.

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