

A Comparison of Channel Estimation between Deep Learning-Based Method and Traditional Pilot-Based Method in OFDM Systems

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Abstract

In an OFDM system, the multipath channel introduces time varying and frequency selective properties in the OFDM symbols, causing Inter-Carrier Interference (ICI) and frequency and phase offset. Thus, channel estimation is imperative for OFDM systems. In most present work, the channel estimation is implemented with merely the traditional pilot-based method. However, this paper has made a comparison under Rayleigh multipath channel between the traditional pilot-based methods including Least Square (LS) and Minimum Mean Square Error (MMSE). We also compare the emerging Machine Learning (ML)-based method and traditional method in the similar multi-path channels. Further impacts of the number of Layers for the deep neural network (DNN) as well as the number of pilots are studied in ML-based channel estimation. The effects of different mapping schemes are also studied within the traditional LS and MMSE method. Besides, a joint frequency offset estimation is provided, regarding the impacts of Cyclic prefix (CP) length and SNR on the Mean Square Error (MSE) of frequency offset estimation. Results have shown that the ML-based method can achieve a relatively good result comparing with traditional method in the similar channels. However, the ML based method has shown a much greater computational complexity than the traditional method, and as the SNR rises, the ML based method tends to have a worse performance than the MMSE method.

Keywords

Channel Estimation; OFDM; Deep Learning; Frequency Offset.

1. Introduction

OFDM (Orthogonal Frequency Division Multiplexing) is a special multi-carrier modulation technology, which uses the orthogonality between carriers to further improve the spectrum utilization, and can resist narrow-band interference and multipath fading. The basic principle of OFDM technology was proposed several decades ago, but due to the limitations of the device level at that time, the application was greatly restricted. In recent years, with the development of technology and device level, as well as the requirements for high speed and reliable transmission, the application of OFDM technology has become more and more widespread. In wireless broadband access and fourth-generation mobile communications (namely 4G cellular standards such as LTE and LTE-A), OFDM technology has become another core technology after the CDMA technology.

The basic principle of OFDM is to split a high-rate data stream into a number of low rate streams, which leads to eliminating Inter Symbol Interference (ISI). The mobile channel possesses a time varying behavior in the received signal energy. The fading amplitudes can be modelled by a Rician or a Rayleigh distribution in a multipath channel, depending on the presence of line of sight signal component. This paper concentrate mainly on the channel estimation in a Rayleigh channel with no line of sight. Generally, the fading channel of OFDM systems can be viewed as a two-dimensional structure (namely in both the time and the frequency domain). However, such 2-D structure is too complicated for practical implementation.

For more common use, channel estimations are usually adopted in OFDM systems. Widely used channel estimation methods mainly include the blind channel estimation and pilot-based channel estimation. The blind channel estimation method requires large amounts of data and has a relatively low convergence rate. Thus, it usually goes against the practical channel estimation.

The pilot-based channel estimation is more widely used nowadays, where the transmitted signal(pilots or preambles) is known at the receiver. Mainly two types of pilots are included: the block pilot type and the comb pilot type. In the block pilot type, all subcarriers of an OFDM symbol are distributed a set of known pilots. [1]

Since the pilot signal is assigned to a particular OFDM block, and sent periodically in the time-domain, this type of pilot arrangement is especially suitable for slow-fading multipath channel. Besides, channel interpolation in the frequency domain is not required, and it is relatively insensitive to frequency selective channels. In the comb pilot mode, only a few subcarriers are used for the initial channel estimation process. Since training sequences are evenly distributed in a whole OFDM block estimation, this type is more suitable for a fast-fading multipath channel. The channel response is usually obtained by LS or MMSE estimation of training sequences.[2]

As technology improves, the traditional channel estimation methods mentioned above can no longer satisfy our need in some specific occasions. Researchers are now paying much attention to implement more heuristic channel estimation techniques for different systems. Among heuristic approaches for channel estimation, Machine Learning(ML) based channel estimation is a promising approach. As is known, ML has been popular recently and used in various fields, such as speech recognition, image processing and so on. However, applying it into wireless communication has not done really well yet. As a result, it is truly valuable to have a try at this aspect. In this paper, we mainly want to make a new approach to do the channel estimation by using deep learning and artificial neural networks in the OFDM system.

To better understand the character of the deep learning based on OFDM system. We found a paper [3] utilizes deep learning for channel estimation and signal detection in OFDM systems. It shows out why the technique used in the channel equalization before can not be directly utilized in channel estimation and then they give out a method to address the issue. Their new approach is to train a DNN model that predicts the transmitted data in diverse channel conditions and the model is used in online deployment to recover the transmitted data.

Besides, we make an attempt to add frequency offset in our communication system in order to find out how it will effect the BER. We use CP of every symbol to estimate the frequency offset based on maximum likelihood method. We make to system with hundreds of subcarriers to transmit the symbols and average every frequency offset to make the estimation. After adding the frequency offset, the system will be more similar to practical wireless communication system.

2. OFDM System and Multipath Channel Description

At the beginning of the system, the binary data stream consisting of both data and pilots are sent to the transmitter. In the following system, $X(0), X(1), \dots, X(N-1)$ are the modulated input data, which form an OFDM block. The data are then converted into parallel data for Inverse Fast Fourier Transform operation(IFFT). The IFFT process then maps $X(0), X(1), \dots, X(N-1)$ onto the N subcarriers and maintains orthogonality of each subcarrier. [4] $X(0), X(1), \dots, X(N-1)$ are the processed data after

IFFT. To remove the ISI and Inter Block Interference(IBI), the guard interval(namely the cyclic prefix) denoted by $X(NG + 1) \dots X(N1)$ with length G are inserted at the start of each block. Later, the parallel data are transformed into parallel data and sent into the channel. [5]The transmitter side is as follows in Figure 1.

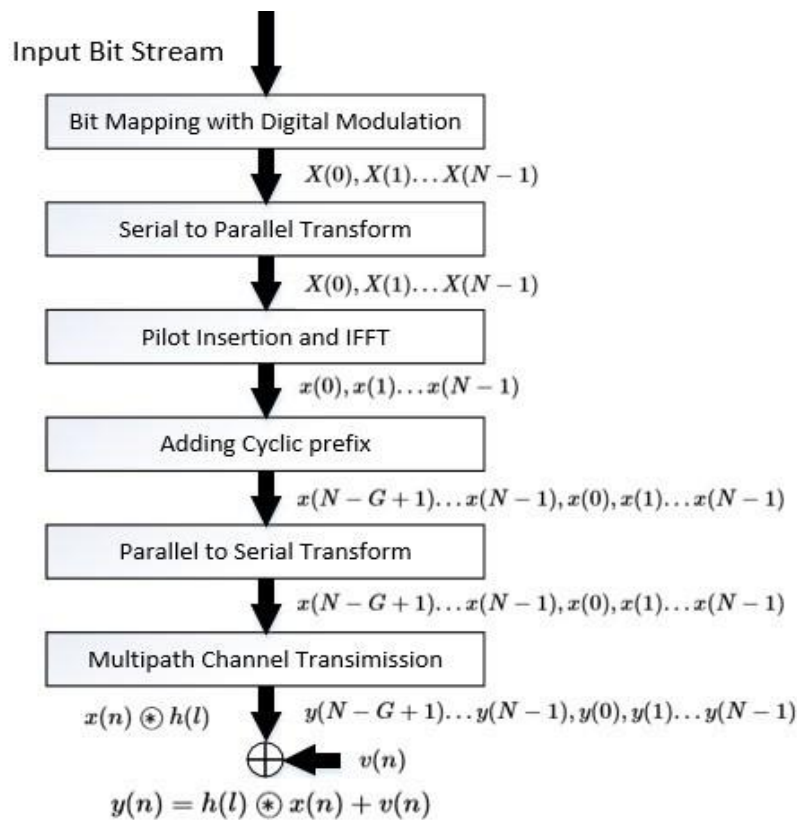


Figure 1: Schematic of OFDM Transmitter

Consider there are L paths in the multipath. Then the serial data are convolved with the channel tap represented by $h(0), h(1) \dots h(L-1)$. Then the zero-mean Additive White Gaussian Noise(AWGN) represented by $v(0), v(1) \dots v(N-1)$ is added. We assume that the time and frequency synchronization is perfect at the receiver side, then the samples on the receiver side can be represented by:

$$y(n) = h(l) * x(n) + v(n) \tag{1}$$

or in matrix form:

$$Y = H^c X + V \tag{2}$$

Where Y is the received sample matrix, X is the transmitted data matrix, V corresponds to AWGN matrix, and H^c is the cyclic channel matrix.[3]

$$h(k) = \sum_{l=0}^{L-1} a_l \delta(k-l) \quad k = 0, 1 \dots L-1 \tag{3}$$

Here, the multipath channel taps (namely the channel pulse response(CIR)) can be denoted by:

where a_l is a zero-mean complex Gaussian random variable with $E[a_i * a_j] = 0$ when ij , denoting the attenuation factor.[3] a_l can also be written as:

$$a_l = P + Q_j \tag{4}$$

where P and Q denotes the real and imagine part and we get $R = P^2 + Q^2$. Presuming the variance of channel coefficients σ_c^2 is known, then the density function of R can be denoted by a Rayleigh distribution:

$$f(R) = \frac{R}{\sigma_c^2} e^{-\frac{R}{\sigma_c^2}} \tag{5}$$

Then comes to the receiver side, the serial received samples are converted to parallel data for further FFT. After the removal of CP, the FFT process are given by:

$$Y(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n) e^{-\frac{j2k\pi n}{N}} \quad k = 0, 1 \dots N - 1 \tag{6}$$

$$H(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} h(l) e^{-\frac{j2k\pi n}{N}} \quad k = 0, 1 \dots N - 1$$

$$V(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} v(n) e^{-\frac{j2k\pi n}{N}} \quad k = 0, 1 \dots N - 1$$

Here, Y(k), H(k) and V(k) respectively represent the output of data samples, channel parameters and AWGN. Represented by matrix form:

$$Y = HX + V \tag{7}$$

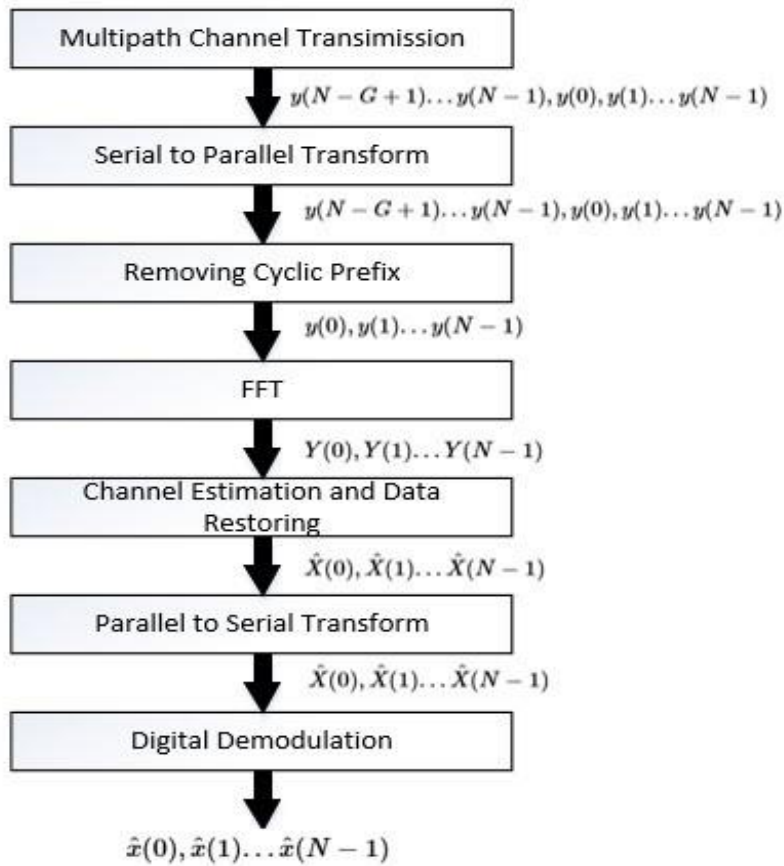


Figure 2: Schematic of OFDM Receiver

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ \vdots \\ Y(N-1) \end{bmatrix} = \begin{bmatrix} X(0) & 0 & 0 \\ 0 & X(1) & 0 \\ 0 & 0 & 0 \\ \vdots & \dots & \vdots \\ 0 & 0 & X(N-1) \end{bmatrix} \begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ \vdots \\ H(N-1) \end{bmatrix} + \begin{bmatrix} V(0) \\ V(1) \\ V(2) \\ \vdots \\ V(N-1) \end{bmatrix} \tag{8}$$

The receiver side is as follows in Figure 2.

3. Method Description

3.1 Traditional method

This paper concentrates on a slow fading Rayleigh channel, where the block type pilots are adopted and inserted in the frequency domain for the further LS and MMSE estimator. Here each pilot symbol are transmitted prior to S_i symbols.[6] The pilot pattern are shown in Figure 3.

3.1.1. LS Algorithm

According to equation(7), the objective cost function $C(\hat{H})$ can be denoted as:

$$C(\hat{H}) = \|Y - X\hat{H}\|^2 = Y^H Y - Y^H X\hat{H} - \hat{H} X^H Y + \hat{H}^H X^H X \hat{H} \tag{9}$$

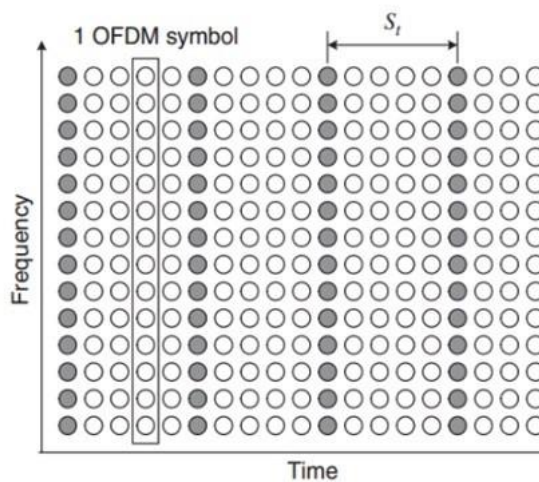


Figure 3: Pattern of Block Type Pilots

Here \hat{H} represents the result of LS channel estimation. We can minimize $C(\hat{H})$ with its partial derivative to \hat{H} , denoted as:

$$\frac{\partial C(\hat{H})}{\partial \hat{H}} = -2(X^H Y)^H + 2(X^H X \hat{H})^H = 0 \tag{10}$$

Then we get

$$\hat{H}_{LS} = (X^H X)^{-1} X^H Y \tag{11}$$

The LS algorithm shows great simplicity and without knowing any knowledge of the statistics of the channels, and thus, is calculated with low complexity.

3.1.2. MMSE Algorithm

Different from LS algorithm, MMSE algorithm aims to get the minimum Mean Square Error(MSE). In this process, we use statistical variables, and we can take the influence of noise into consideration.

The definition of the Mean Square Error is:

$$MSE = E[(H - \hat{H})(H - \hat{H})^H] \tag{12}$$

H and \hat{H} represent the true value and the estimated value of the channel response. MMSE algorithm aims to find a matrix G, to make GY be close to H as much as possible. Replace \hat{H} by GY and take partial derivative of MSE with respect to G, we can find when G equals to $R_{hy}R_{yy}^{-1}$, the minimum MSE is gotten. So we get:

$$\hat{H}_{MMSE} = R_{hy}R_{yy}^{-1}Y \tag{13}$$

R_{hy} represent the cross correlation matrix of the channel response and the received signal,

R_{yy} represent the autocorrelation matrix of the received signal.

Then we analyze R_{hy} and R_{yy} respectively and express them by R_{hh} , the autocorrelation matrix of channel response. We can get:

$$R_{hy} = E(HY^H) = E[H(XH + N)^H] = R_{hh}X \tag{14}$$

and

$$R_{yy} = E(YY^H) = E[(XH + N)(XH + N)^H] = XR_{hh}XH + \sigma^2I \tag{15}$$

So we get:

$$\hat{H}_{MMSE} = XR_{hh}(XR_{hh}XH + \sigma^2I)^{-1}Y = R_{hh}[R_{hh} + (XX^H)^{-1}\sigma^2I]^{-1}\hat{H}_{LS} \tag{16}$$

We can find that the estimated value of MMSE algorithm is based on the estimated value of LS algorithm. We take the influence of noise into consideration and also need to know certain information of the channel.

3.2 Deep learning based method

Deep learning is one kind method of machine learning method. Usually it's used to demonstrate some black boxes. We can divide the whole process of deep learning into two parts: forward propagation and backward propagation. For example, we have a few known information called $Y(k)$, we want to obtain the unknown information $X(k)$ from $Y(k)$, but we don't know what happened in the actual process. We just input $Y(k)$ into a neural network, and the output is called $\hat{X}(k)$. Then we can treat $\hat{X}(k)$ as an estimation of $X(k)$. This is the forward propagation.

However, it is definitely not just a random network that can give us a good estimation, so we need to do the training. We need to have many known $X(k)$ as the label of training, by comparing the difference between $\hat{X}(k)$ and $X(k)$, we can do the back propagation to optimize the neural network. After many times of training like this, we can get a good estimation of $X(k)$.

Specifically, in the OFDM system, we treat the recovered data as $Y(k)$ and transmitted data as $X(k)$, and then we can get the estimation of transmitted data. So we can see that we don't obtain the specific channel estimation in this process, so the channel is like a black box in the deep learning network.

The model we choose here is MLP, which is the same as the model used in paper[2]. MLP means Multilayer Perceptron(Figure 4). We can see that every line is the linear transformation of last layer.

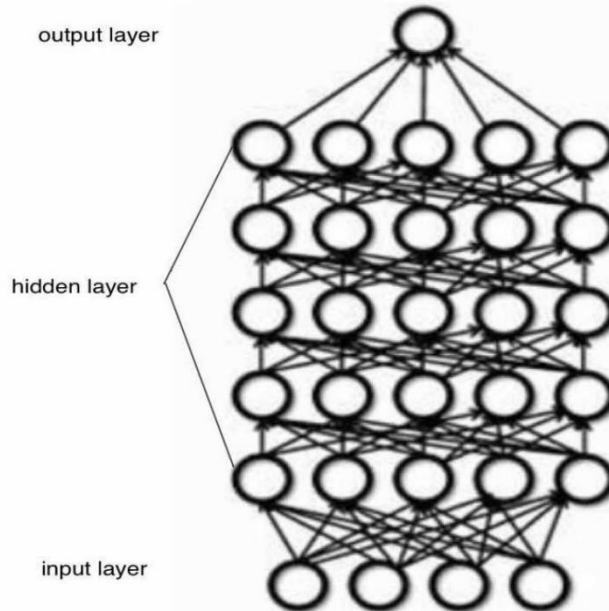


Figure 4: A deep learning model.

According to the calculation of the output, it can be found that each neuron of the hidden layer is actually composed of a linear combination of the input $Y(k)$. However, if it is only a linear combination, then no matter how many layers this neural network has, the result will be linearly

related to the $Y(k)$. So we add an activation function after each neuron result z to change the linear rule, such as using the sigmoid function, the formula is as follows:

$$f(x) = \frac{1}{1+e^{-x}} \tag{17}$$

and the relu function:

$$f(x) = \max_{x \in R}(0, x) \tag{18}$$

In the back propagation, to reduce the loss function, the strategy we choose is the Gradient Descent. Its gist is that we advance a small step in the direction of the fastest decline of the function, namely the opposite gradient direction. After many steps, we can reach the lowest point of the loss function finally. The loss function we choose is the MSE.

$$F(\hat{X}[k], X[k]) = \sum_{k=1}^N \frac{|\hat{X}[k]-X[k]|^2}{N} \tag{19}$$

Like the traditional method, we also set up 64 subcarriers. In this case, for one transmission, we generate 128 bits($X(k)$) to transmit. But when the output is the predication of all of transmitted data (128bits),namely we want to get 128 bits at one time, it'll take a long time to train, because our computer is not very powerful. Therefore we choose to predict 16 bits every time, namely I predict in 8 times. In this case, the loss function converges much faster.

3.3 Frequency Offset

Frequency offset is a common phenomenon in communication system and it has something to do with channel estimation. When there exists frequency offset in a communication system, the received symbols are different from those which are transmitted in no frequency offset system. So it is important to do frequency offset estimation to demodulate received symbols more correctly. The frequency offset is generated because the symbols will be sent to high frequency part when transmitting and they will be sent back to baseband when arriving the receivers. Different systems have different offset. In the paper, we put in a 0.25 frequency offset and make an estimation. We use maximum likelihood method to make the estimation. The following is algorithm. First, we make the first two symbols in every frame with the same prefix to form a cyclic prefix. Then all the symbols in one subcarrier will be transmitted in the channel and at the receiver part we make a comparison between the first symbol and the second symbol's prefix to get a frequency offset. In the next step, we average frequency offset in all the subcarriers and use maximum likelihood method to get the MLE.[7]

$$y = r_n + w_n \tag{20}$$

$$r_n = \left(\frac{1}{N}\right) \sum_{k=-K}^k X_k H_k e^{j2\pi n(k+e)/N} \quad n = 0,1 \dots 2N - 1 \tag{21}$$

$$r_{1k} = \sum_{n=0}^{N-1} r_n e^{-j2\pi kn/N} \quad k = 0,1 \dots N - 1 \tag{22}$$

$$r_{2k} = \sum_{n=N}^{2N-1} r_n e^{-j2\pi kn/N} = \sum_{n=0}^{N-1} r_{n+N} e^{-j2\pi kn/N} \quad k = 0,1 \dots N - 1 \tag{23}$$

$$\varepsilon = \frac{1}{2\pi} \tan^{-1} \left[\frac{\sum_{k=0}^{N-1} I_m(Y_{2k} * Y_{1k}^3)}{\sum_{k=0}^{N-1} Re(Y_{2k} * Y_{1k}^3)} \right] \tag{24}$$

(20) y represents received symbol

(22)represents the K point in the first received prefix

(23)represents the K point in the second received prefix

(24)represents the estimated frequency offset

4. Simulation Results

4.1 Comparison between LS and MMSE

First, the MSE and BER is compared between the traditional LS Algorithm and MMSE Algorithm under 4-QAM modulation scheme. The SNR range has been chosen from 10 to 25. As shown in

Figure 5 and Figure 6, MMSE Algorithm performs considerably better than LS Algorithm under the given SNR, while LS Algorithm has even higher MSE in lower SNR environment.

Table 1: Simulation Parameters for Traditional Channel Estimation

Parameters	Specifications
Number of Sub-carriers	64
FFT Size	64
Cyclic Prefix Length	16
Cyclic Prefix Ratio	1/4
Fading Channel	AWGN, Rayleigh
Number of Multi-path	16
Modulation Technique	4-QAM, 16-QAM

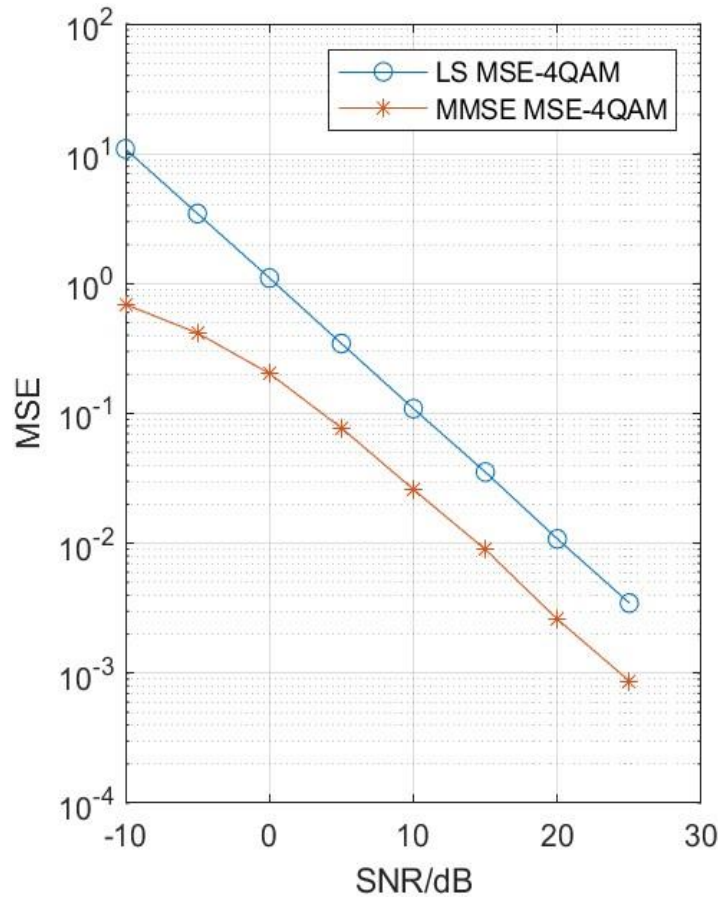


Figure 5: MSE of LS and MMSE

4.2 Comparison between Different Mapping techniques

To testify the advantage of MMSE over LS under all conditions, a comparison has been given between 4-QAM modulation and 16-QAM modulation. As is shown in Figure 7, both the LS and MMSE shows higher BER in 16-QAM than in 4-QAM, while MMSE shows stable advantage over LS in either modulation scheme.

4.3 Impact of Number of Layers

We want to observe how different neural network layers affect the results. We set SNR to 10db, the activation function of the first few layers is relu function, and the activation

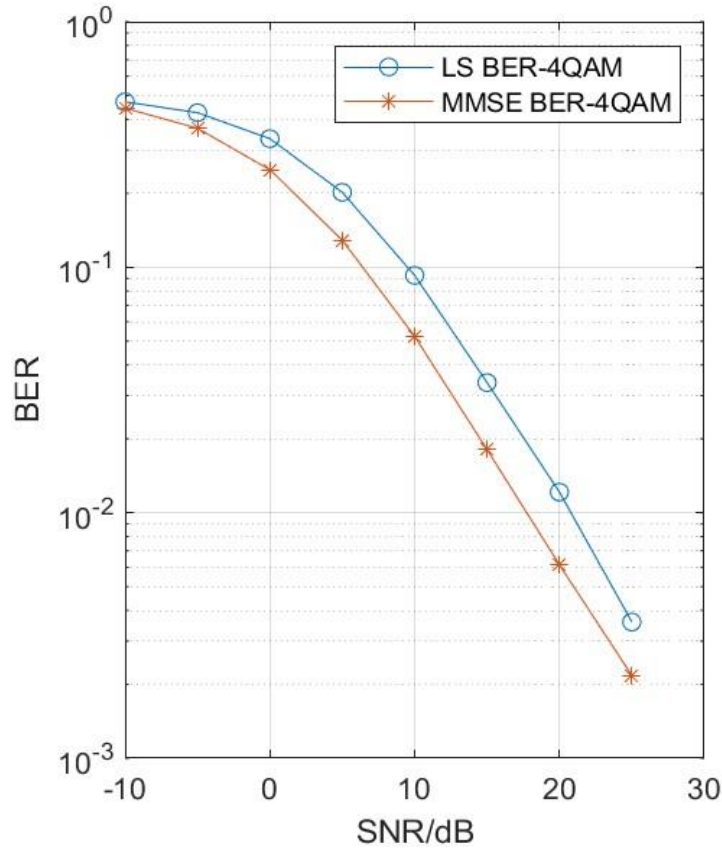


Figure 6: BER of LS and MMSE

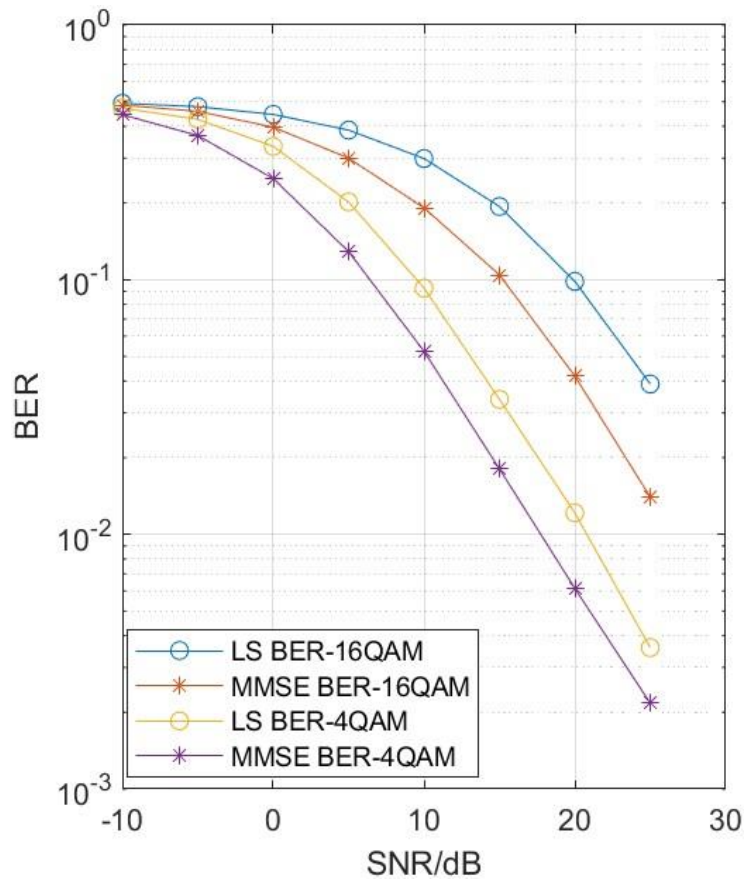


Figure 7: BER of LS and MMSE under 4-QAM and 16-QAM function of the last layer is the sigmold function.

Table 2: Impact of Number of Layers

number of layers	structure	BER
2	256-16	0.15
3	256-120-16	0.07
4	256-500-120-16	0.042
5	256-500-250-120-16	0.031
6	256-500-250-120-32-16	0.030

The result shows that the performance we get is relatively poor when the number of layers of the neural network is too small, and when the number of layers gradually becomes larger, the effect will gradually become better. Despite this, it does not mean that more layers can give us better performance. Too many layers will cause the overfitting of network and also take up too many resources. Finally, we choose 5 layers structure.

4.4 Comparison between Traditional and Deep Learning Based Method

From Figure 8, we can see that the performance of deep learning is slightly better than traditional method when we insert the same 64 pilots. However, the dataset we used to do the training and testing is a little different from the one we used in the traditional method. In the traditional method, the dataset we used to do the testing is generated by ourselves; in the deep learning part, we use the dataset generated by paper[2], so maybe it's not a fair comparison. We can't conclude that the deep learning model can bring us more accurate results than the traditional methods. But the channels we use in these two parts are similar, they are all multi-path channels. At least we can conclude that deep learning based method is feasible, because it can achieve a relatively good performance compared with traditional methods in the similar channels.

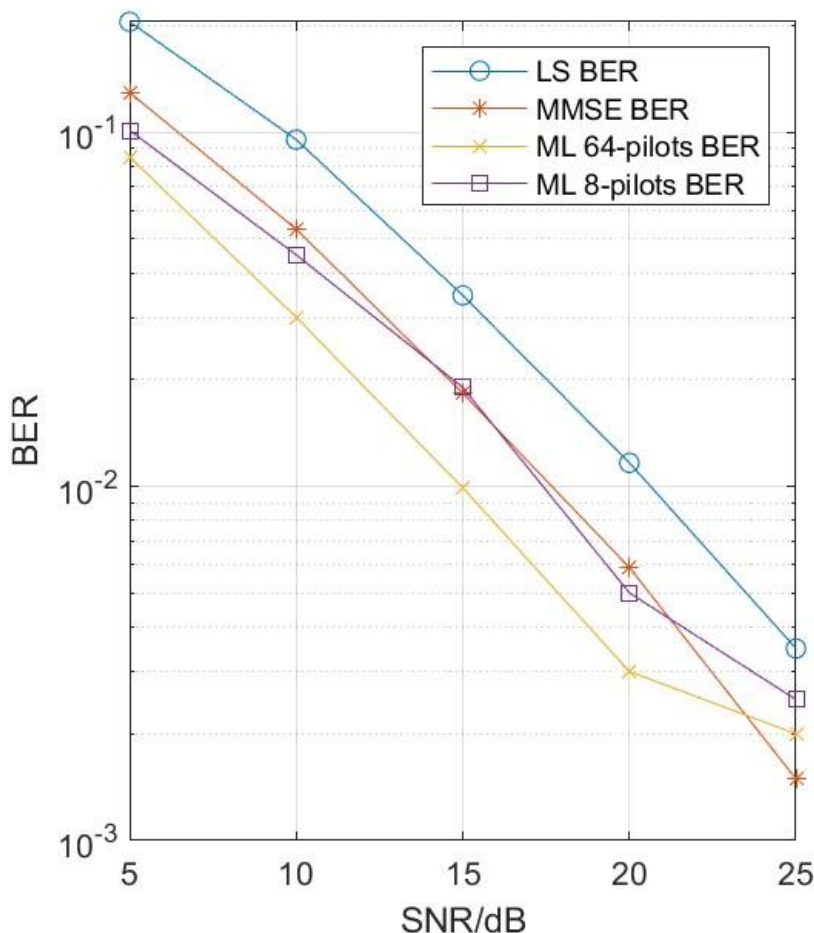


Figure 8: BER of LS,MMSE and Machine Learning (ML).

4.5 Impact of Different Pilots

The impact of pilots is a very important effect that needs to be considered. Even if we insert pilots, the network does not utilize the information of the pilots directly. Then we compare the result of using pilots with not using. We set SNR to 20db, after 100 epoch, BER of no pilots can achieve 0.23, and BER of 64 pilots can achieve 0.003. After the training of same epoch, 64 pilots is much better. However, it does not mean that the final result without pilots must be so much worse than that with pilots, because the results without pilots have not converged in this case. We can only conclude that the less pilots we use, the more time we need to do the training.

Considering it would take a long time to train without pilots, we compared the performance of using 64 pilots and 8 pilots. We can also see the result from Figure 8. And we found that BER of 64 pilots is slightly lower than 8 pilots. Maybe it's related to the input of the network. There will be more information of the input of the network fixed when we use more pilots, so it can make a difference to the final result. Another explanation we guess is that the pilots will make the loss function converge faster as known information, namely 8 pilots network does not get enough training. Perhaps when we train it for more time, it can also perform as good as 64 pilots, so we do more training on the 8 pilots. However, we found that the change is very slight. So we think maybe the difference is more related to the influence on the final converged result caused by the structure of input.

4.6 Frequency Offset

After combining the OFDM system and maximum likelihood algorithm, we change the added noise and length of CP to work out the simulation, which shows how MSE of frequency offset changes.

4.6.1. MSE changes with different SNR

From the figure 9, we can find out that MSE will be smaller with the SNR becoming bigger and bigger. The three curves seems to be straight lines meaning that MSE decreases smoothly with SNR increasing. Different colors represent different lengths of the CP.

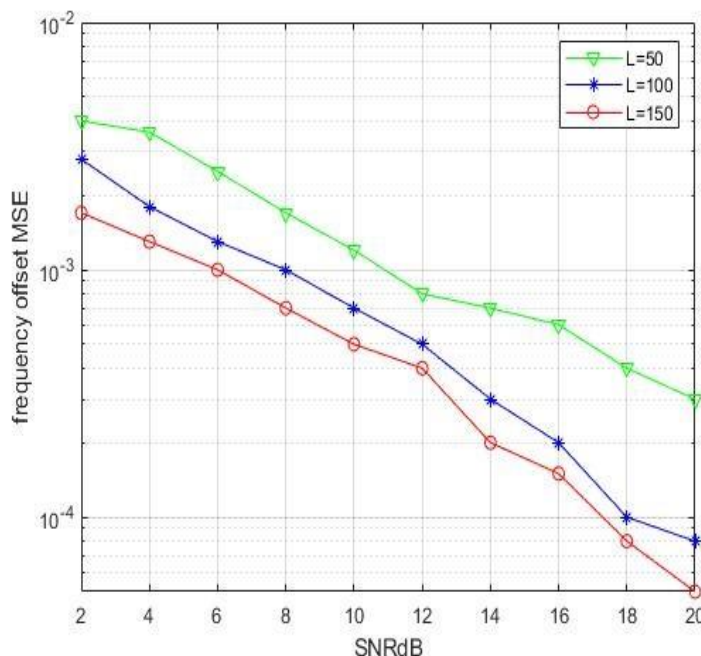


Figure 9: MSE of offset frequency with different SNR.

4.6.2. MSE changes with different length of CP

From the figure 10, we can find out that MSE will get smaller with the length of CP bigger and bigger. The three curves have fluctuations with the CP increasing, but the trend is that MSE continues decreasing.

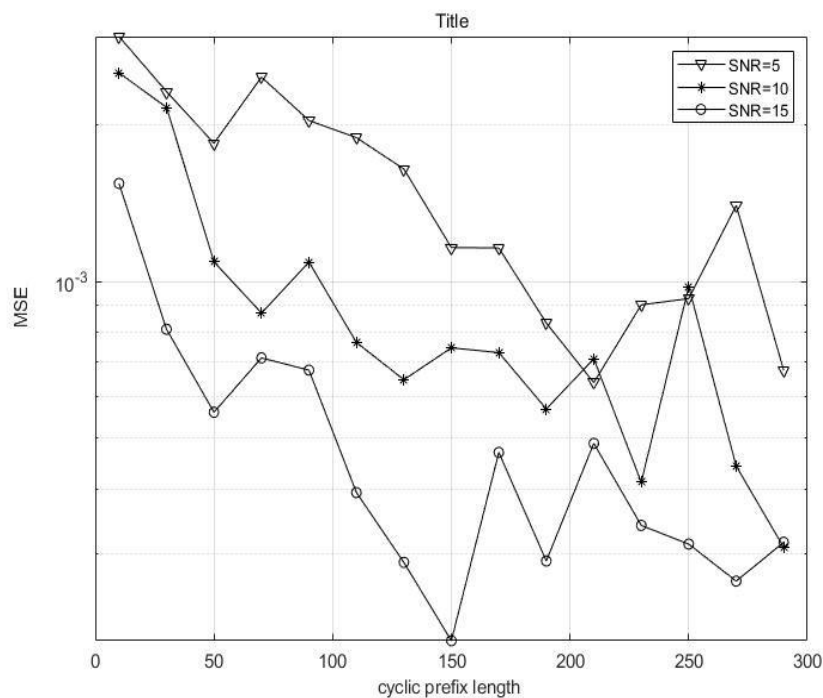


Figure 10: MSE of offset frequency with different CP length.

5. Conclusions

In this paper, a brand-new comparison of OFDM system channel estimation is given, and knowledge regarding both the burgeoning ML-based channel estimation and traditional pilot-based channel estimation has been provided.

In the traditional part, while the LS Algorithm tends to be the simplest, its highest BER remains unbearable in practical use. The MMSE Algorithm requires more data than LS, including the covariance of noise and basic data of the Rayleigh channel, but performs considerably better than LS. But considering that the MMSE doesn't cause too much calculation complexity, it is a more preferable method for traditional use. And as the order of digital modulation increases, the BER increases greatly, but MMSE exhibits better performance than LS consistently.

In the deep learning part, we have obtained some knowledge and experience about using deep learning model. For example, we found that the selection of the number of network layers should not be too small, nor too big. 5 layers is enough in this case.

We can also see the potential and drawback of deep learning in wireless communication. We found that the performance of deep learning is slightly better than traditional method. However, it's not a fair comparison, though we do experiments in the similar channel. Additionally, the deep learning method is actually trained on a specific dataset. This prediction is actually limited and requires a lot of training. When new channels are added, they may have a great impact on the results. So it needs to be based on a great deal of data. Compared with traditional method, we also conclude that deep learning based method is not very dependent on pilots, which is its advantage.

Then comes to the frequency offset part, offset frequency is very dependent on SNR and the length of CP. In our wireless system, working out the estimation of frequency offset will help us get more accurate symbols and reach a lower BER.

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