

# Morphological Analysis of the Combination Structure of boom-forestay of the crane Based on Transfer Matrix Method

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## Abstract

In order to study the influence of the upper forestay-hinge structure on the boom of the crane, the forestay is considered as the elasticity, and the static balance model of boom-forestay structure of the crane is established. The transfer matrix method is used to derive the equilibrium equation of the whole structure, and the problem of solving the nonlinear equations is transformed into the optimization problem. Taking a 50t crane as the research object, the various conditions of the position change of the equivalent load on the boom are simulated, and the orthogonal array analysis method is used to solve the calculation target extremum, and the structural optimization problem is solved. Based on this compared with other methods, the feasibility of this method is verified. The results show that the complete transfer matrix method, which takes into account the upper forestay structure of the crane, can effectively deal with the stress analysis of the boom-forestay structure.

## Keywords

Crane; transfer matrix; orthogonal design; Orthogonal array.

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## 1. Introduction

At present, when constructing the boom model of the crane, the rod of the forestay is simplified to a spring, and the equivalent stiffness support is used instead of the rod; or the force method is used to solve the stress of the rod. The effect of the hinge structure on the stress distribution of the boom is not considered. Shen Jian et al. modeled the overall structure, using the boom and forestay as rigid bodies, using the principle of minimum potential energy to optimize the length of the rods, and calculating the inclination of the boom when the trolleys were in different positions. In order to calculate the stress distribution of the boom of the crane more accurately, the model has the following improvements:

- a) Consider the influence of the forestay on the boom.
- b) All bars are considered elastic.
- c) Consider the trolley and hoist as a concentrated load on the boom.

In this paper, the transfer matrix method is used to establish the equilibrium equation of the boom of the crane before the load, and the stress distribution of the equivalent load at different positions is calculated. Compared with other models, the influence of the hinge structure of the forestay on the crane is analyzed, which lays a foundation for the safety and reliability design of the structure.

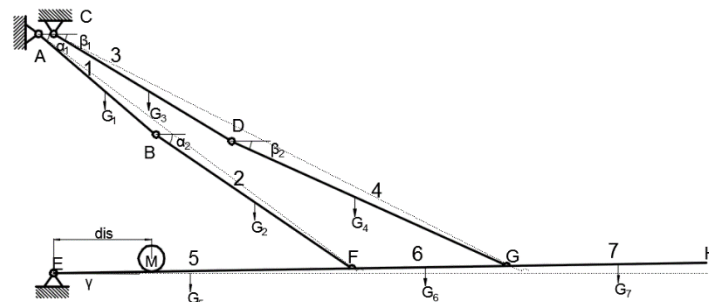


Figure 1. Simplified model of the crane

## 2. Basic theory of transfer matrix

The transfer matrix method is a simple calculation method for static, dynamic and stability analysis of engineering structures. It has the characteristics of high precision, clear mechanical concept and easy programming. Flexible in use and high in computational efficiency, it is widely used in modern engineering.

Figure 2 shows the force relationship at both ends of a beam, and the mass of the beam is equalized and applied to the beam. The direction is along the axis direction and perpendicular to the axis direction.

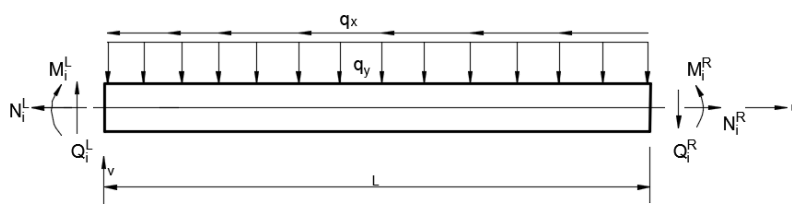


Figure 2. Equal beam with uniform load

The uniform stress of the unit line of the beam is ( $G$  is the gravity of the beam, and  $\alpha$  is the angle between the gravity and the beam):

$$q_x = \frac{G \cdot \sin \alpha}{L} \tag{1}$$

$$q_y = \frac{G \cdot \cos \alpha}{L}$$

As shown in Figure 2, for any segment of the beam, the transfer relationship between the left and right state vectors is as follows:

$$\begin{bmatrix} u \\ v \\ \theta \\ M \\ N \\ Q \\ 1 \end{bmatrix}_i^R = \begin{bmatrix} 1 & 0 & 0 & 0 & L/(EA) & 0 & -q_x L^2/(2EA) \\ 0 & 1 & L & L^2/(2EI) & 0 & L^3/(6EI) & -q_y L^4/(24EI) \\ 0 & 0 & 1 & L/(EI) & 0 & L^2/(2EI) & -q_y L^3/(6EI) \\ 0 & 0 & 0 & 1 & 0 & L & -q_y L^2/2 \\ 0 & 0 & 0 & 0 & 1 & 0 & -q_x L \\ 0 & 0 & 0 & 0 & 0 & 1 & -q_y L \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ \theta \\ M \\ N \\ Q \\ 1 \end{bmatrix}_i^L \tag{2}$$

It is denoted as:

$$Z_i^R = U_i Z_i^L \tag{3}$$

Where  $Z$  is the state vector and  $U$  is the transfer matrix.  $E$  is the modulus of elasticity,  $A$  is the cross-sectional area, and  $I$  is the moment of inertia. The effect of the axial load is considered in the matrix. For the hinged beam shown in Figure 3, the transfer matrix at the hinge point is:

$$T_{arr} = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 & 0 & 0 & 0 & 0 \\ -\sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 0 & 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Where  $K = \frac{\theta_{i+1}^L}{\theta_i^R}$ ;  $\theta_i^R = \frac{q_i L_i^3}{24EI_i}$ ;  $\theta_{i+1}^L = -\frac{q_{i+1} L_{i+1}^3}{24EI_{i+1}}$ , (Do not consider the effects of vertical and horizontal bending). The transfer of the corners is quantized into a fixed ratio in the matrix.

When passing from one component to the next, you need to convert the coordinates. The coordinate transformation matrix is:

$$T = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 & 0 & 0 & 0 & 0 \\ -\sin \alpha & \cos \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 0 & 0 & -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

In the model, the influence of the trolley and the hoisting weight is considered as the concentrated load acting on the beam, as shown in Figure 4:

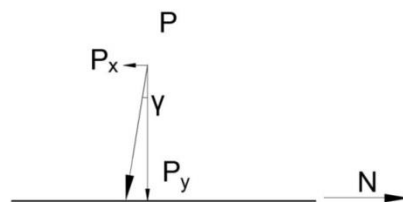


Figure 4. Concentrated load action

Its point transfer matrix is:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & P_x \\ 0 & 0 & 0 & 0 & 0 & 1 & -P_y \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

### 3. Establishment of the front beam-pull rod model of crane

Figure 1 shows a simplified model of the boom-forestay structure of crane. In this paper, each forestay is divided into two sections, numbered in the whole order, and consists of 7 components. They are boom L5, L6, L7, outer forestay L3, L4, and inner forestay L1, L2. The spreader and trolley are

simplified to the equivalent load M. Assuming that the mass of each part is uniformly distributed, and ignoring the influence of friction at the hinge point and longitudinal and transverse bending, the gravity of each part is equivalent to uniform distribution for calculation. Under the joint action of self-weight and load, the structure is adjusted from the initial state to the geometrically stable state, and then the elastic deformation of the rod is coordinated at the hinge point to finally reach the static balance. When the equation is established, the coordinate system is established along the axis and the vertical axis direction on each component, and when it is passed to the next component, the transformation matrix is used to coordinate the coordinate system.

The dotted line in Fig. 1 is the initial state position, and the solid line is the balanced position. The coordinates of the shore bridge are A(0,0), C(0,0), E(2,-24). Initial position, (Animation of angle: based on the level, counterclockwise is positive, clockwise is negative).

The boundary conditions are:

$$\begin{aligned} Z_1^L &= [0 \ 0 \ \theta_1 \ 0 \ N_1 \ Q_1 \ 1]^T; \\ Z_3^L &= [0 \ 0 \ \theta_3 \ 0 \ N_3 \ Q_3 \ 1]^T; \\ Z_5^L &= [0 \ 0 \ \theta_5 \ 0 \ N_5 \ Q_5 \ 1]^T; \\ Z_7^R &= [u_7 \ v_7 \ \theta_7 \ 0 \ 0 \ 0 \ 1]^T; \end{aligned} \tag{7}$$

The increased unknown intermediate state vector is:

$$Z_6^R = [u_6 \ v_6 \ \theta_6 \ M_6 \ N_6 \ Q_6 \ 1]^T; \tag{8}$$

There are the following relationships at the nodes:

$$\begin{cases} Z_6^L = Z_5^R + E_1 \cdot Z_2^R \\ E_2 \cdot Z_6^L = E_2 \cdot Z_2^R \\ E_3 \cdot Z_2^R = 0 \end{cases}, \begin{cases} Z_7^L = Z_6^R + E_1 \cdot Z_4^R \\ E_2 \cdot Z_7^L = E_2 \cdot Z_4^R \\ E_3 \cdot Z_4^R = 0 \end{cases} \tag{9}$$

Convert to matrix form:

$$\begin{aligned} Z1 &= TF \cdot U_2 \cdot TB \cdot U_1; \\ Z2 &= TG \cdot U_4 \cdot TD \cdot U_3; \\ UA &= \begin{bmatrix} U_6 \cdot E_1 \cdot Z1 & 0_{7 \times 7} & U_6 \cdot U_5 & -I_7 & 0_{7 \times 7} \\ E_2 \cdot Z1 & 0_{2 \times 7} & -E_2 \cdot U_5 & 0_{2 \times 7} & 0_{2 \times 7} \\ E_3 \cdot Z1 & 0_{1 \times 7} & 0_{1 \times 7} & 0_{1 \times 7} & 0_{1 \times 7} \\ 0_{7 \times 7} & U_7 \cdot E_1 \cdot Z2 & 0_{7 \times 7} & U_7 & -I_7 \\ 0_{2 \times 7} & E_2 \cdot Z2 & 0_{2 \times 7} & -E_2 & 0_{2 \times 7} \\ 0_{1 \times 7} & E_3 \cdot Z2 & 0_{1 \times 7} & 0_{1 \times 7} & 0_{1 \times 7} \end{bmatrix}; ZA = \begin{bmatrix} Z_1^L \\ Z_3^L \\ Z_5^L \\ Z_6^R \\ Z_7^R \end{bmatrix} \end{aligned} \tag{10}$$

The optimization parameters are the five angular values of the structure. After determining the structural configuration, the stress distribution of the structure is calculated after solving the boundary conditions. When solving the boundary conditions, pay attention to the 7th and 17th behavior extensions in the matrix UA, and remove the extensions when solving. The above formula is transformed into a form that expresses the unknown by matrix multiplication and division, and is solved by numerical calculation, thereby solving the boundary condition.

#### 4. Example analysis

Taking a 50t crane as an example, the stress distribution of the boom structure under different working conditions is calculated. The domain of the solution space is:

$$\alpha_1^{initial} = \alpha_2^{initial} = -\text{atan}(24000/26600); \beta_1^{initial} = \beta_2^{initial} = -\text{atan}(24000/55500); \gamma^{initial} = 0$$

$$\alpha_1 \in [-0.8 \quad \alpha_1^{initial}]; \alpha_2 \in [\alpha_2^{initial} \quad -0.6]; \beta_1 \in [-0.5 \quad \beta_1^{initial}]; \beta_2 \in [\beta_2^{initial} \quad -0.3]; \gamma \in [0 \quad 0.01]$$

The basic parameters of model of a crane as shown in Table 1 are as follows ( $g = 9.8m / s^2$ ):

Table 1. Structural parameters of boom-forestay

Element	Length (m)	Area (m <sup>2</sup> )	Inertia (m <sup>4</sup> )	Quality (KG)
L1	16.8268	2.34×10 <sup>-2</sup>	1.35045×10 <sup>-4</sup>	3701.9
L2	19	2.34×10 <sup>-2</sup>	1.35045×10 <sup>-4</sup>	4180
L3	28.4669	4.3552×10 <sup>-2</sup>	4.9413×10 <sup>-4</sup>	11102.104
L4	32	4.3552×10 <sup>-2</sup>	4.9413×10 <sup>-4</sup>	12480
L5	24.6	1.36224×10 <sup>-1</sup>	8.1026×10 <sup>-2</sup>	55608.2016
L6	28.9	1.36224×10 <sup>-1</sup>	8.1026×10 <sup>-2</sup>	65328.3344
L7	7	1.36224×10 <sup>-1</sup>	8.1026×10 <sup>-2</sup>	15823.472
M	—	—	—	50000

The solution model is established according to the transfer matrix method and solved by the orthogonal table method. And compared with the results calculated by the force method and the equivalent stiffness method. The stress of the boom is related to the load, structural parameters, and position of the load. Therefore, it is necessary to calculate the bending moment and deflection curve of the front girders under different working conditions.

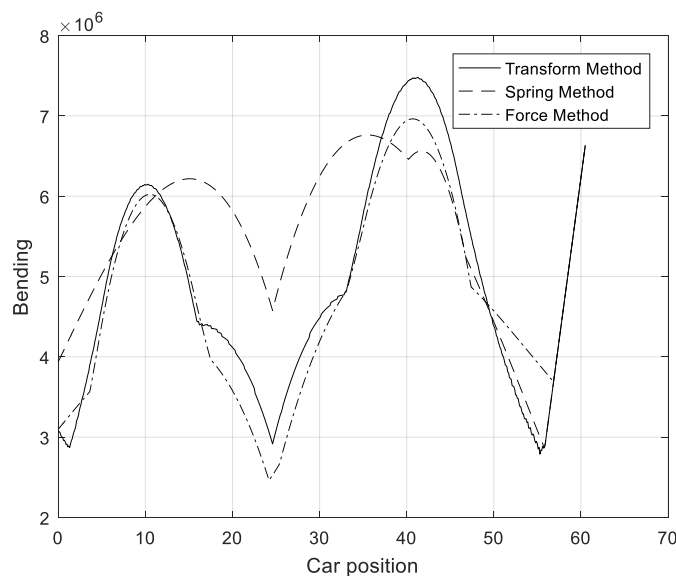


Figure 3. Maximum bending moment envelope of the boom under different operating conditions

Figure 3 is the maximum bending moment envelope of the boom when the load is at different positions. It can be seen that the bending moment diagram of the transfer matrix method is mostly the same as the force method. Especially when the load position is in the middle of the EF section and the FG section, the calculation result of this model is larger than the other two models. When the load is on the GH segment, the curves calculated using the three models are roughly coincident, indicating that the results of the different models have the same effect on the boom when the load is at these locations.

Figure 4 is an axial force diagram of the drawbars L1, L2, L3, L4 when the load is at different positions. It can be concluded that when the load position moves from point E to end point G, the axial forces of L1 and L2 gradually increase, and the axial forces of L3 and L4 remain substantially unchanged, that is, the influence of the load on the inner forestay is gradually increased, and outer forestay is gradually increased. The influence is small; when the load approaches the end point G, the axial force of L1, L2 begins to decrease, and the axial force of L3 and L4 increases rapidly, indicating that the force of the load distribution to the outer forestay increases, and inner forestay begins to reduce the bearing force; At the end of the boom, the axial force of the front struts reaches a peak. Due to the influence of elastic deformation, the axial force on L1 and L2 is smaller than the load at point E.

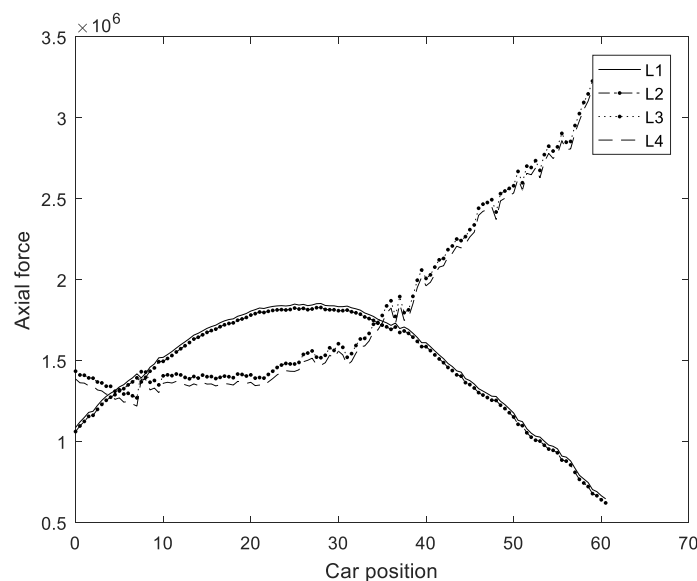


Figure 4. Axial force diagram of the forestay under different conditions

## 5. Conclusion

In the establishment of the boom-forestay model of the crane, a) the forestay hinge structure is considered; b) the bar is an elastic body; c) the axial load is affected. According to the relevant theory, the overall equilibrium equation of the model is derived, and the nonlinear equations are transformed into optimization problems, and the orthogonal array method is used for calculation. The influence of the forestay structure on the internal force distribution of the boom is analyzed when the load is at different positions. The results show the following regularity:

- 1) The bending moments at the position of the F point at the hinge of boom and the inner forestay, the calculation results of the complete transfer matrix method are significantly smaller than the other two methods, that is, the forestay hinge structure is reasonable and correct.
- 2) The internal force distribution in the cantilever area of the boom is not affected by the hinged structure of the forestay.

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