

# Rotary-wing UAV Trajectory Tracking Control Studies

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## Abstract

**This paper analyses and reviews the trajectory tracking control technology of rotary-wing UAVs. Firstly, it introduces the research background and significance of UAV trajectory tracking control, then it introduces the current commonly used UAV modelling methods, then it summarizes the current mainstream trajectory tracking control methods, and analyses and compares their advantages and disadvantages. Then it introduces the common simulation software currently used in the study of trajectory tracking control and the basis for judging the effectiveness of the control method, and gives examples of simulation experiments, and finally introduces the UAV hardware experiment platform. This paper aims to provide a comprehensive overview for research scholars in the field of UAV control to promote the development and application of UAV trajectory tracking control technology.**

## Keywords

**Unmanned Aerial Vehicle; Trajectory Tracking Control; Numerical Simulation.**

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## 1. Introduction

In 1907, the scholar J. Leishman first proposed the concept of quadrotor <sup>[1]</sup> and demonstrated the quadrotor helicopter built by him, and since then, rotary-wing type of flying vehicles have been known to people. And rotary-wing UAVs have greatly stimulated the interest of enterprises, governments, and armies in multi-rotor aerial vehicles due to their advantages such as simple structure, low cost, and wide application areas. As shown in Fig. 1, in the last decade, multi-rotor UAVs have been widely used in various industries, and gradually become an important tool for surveying and mapping, security monitoring, agricultural plant protection, aerial photography, and other tasks <sup>[2]</sup>. However, the key to achieve efficient and precise operation of rotary-wing UAVs in these application scenarios lies in the design of a stable and reliable trajectory tracking control system, which has caused a large number of scholars to carry out in-depth research on the trajectory tracking control technology of rotary-wing UAVs.

Trajectory tracking control refers to the movement of a UAV in space according to a specified or planned trajectory, and automatically adjusts the attitude, thrust and other parameters during the movement to achieve accurate tracking of the target trajectory. In rotary-wing UAVs, due to their complex dynamics and nonlinear characteristics, trajectory tracking control faces many challenges, such as parameter uncertainty, power balance, and wind disturbance effects. Therefore, the study of trajectory tracking control technology for rotary-wing UAVs is of great significance.

The purpose of this paper is to review and analyse the trajectory tracking control technology of rotary-wing UAVs. Firstly, it introduces the common dynamics modelling of rotary-wing UAVs, and then systematically combs through the existing trajectory tracking control methods, which mainly include

linear control and nonlinear control and other techniques, and evaluates and compares the merits and demerits of various methods. Finally, numerical simulation of the common linear control technique PID control as well as its improvement method is carried out, and then the flight experiment hardware platform is introduced. Through the review of this paper, it aims to provide reference and reference for the research and application of trajectory tracking control technology of rotary-wing UAVs.



Fig. 1 Rotary drone application scenarios

## 2. Unmanned Aerial Vehicle Modelling

Using the Eulerian Lagrangian modelling principle, the dynamics of the quadrotor UAV is modelled as follows:

$$m\ddot{\mathbf{p}} = U\mathbf{R}\mathbf{e}_3 - m\mathbf{g}\mathbf{e}_3 + \mathbf{d}_U \quad (1)$$

$$\mathbf{J}\ddot{\boldsymbol{\theta}} = \boldsymbol{\tau} - \mathbf{C}\dot{\boldsymbol{\theta}} + \mathbf{d}_r \quad (2)$$

Eqs. (1) and (2) represent the dynamics models of the position subsystem and attitude subsystem of the UAS, respectively. In the formula,  $\mathbf{p} = [x, y, z]^T$  is the position of the centre of mass of the UAV in the inertial coordinate system;  $\boldsymbol{\theta} = [\phi, \theta, \psi]^T$  is the rotational attitude of the UAV expressed in Euler angles;  $m$  denotes the mass of the drone,  $\mathbf{g}$  denotes the acceleration of gravity;  $\mathbf{e}_3 = [0, 0, 1]^T$  denotes the unit vector in the vertical direction;  $U$  and  $\boldsymbol{\tau}$  are the control inputs to the system and represent the lift and rotational moments, respectively.  $\mathbf{R}$  denotes the translation matrix of the translational velocity from the body coordinate system to the inertial coordinate system with the following expression:

$$\mathbf{R} = \begin{bmatrix} \cos \theta \cos \psi & \sin \theta \sin \phi \cos \psi - \cos \phi \sin \psi & \sin \theta \cos \phi \cos \psi + \sin \phi \sin \psi \\ \cos \theta \sin \psi & \sin \theta \sin \phi \sin \psi + \cos \phi \cos \psi & \sin \theta \cos \phi \sin \psi - \sin \phi \cos \psi \\ -\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta \end{bmatrix} \quad (3)$$

$\mathbf{J}$  is the representation of the UAV rotational inertia in the inertial coordinate system:

$$\mathbf{J} = \begin{bmatrix} I_x & 0 & -I_x \sin \theta \\ 0 & I_y \cos^2 \phi + I_z \sin^2 \phi & (I_y - I_z) \sin \phi \cos \phi \cos \theta \\ -I_x \sin \theta & (I_y - I_z) \sin \phi \cos \phi \cos \theta & I_x \sin^2 \theta + I_y \sin^2 \phi \cos^2 \theta + I_z \cos^2 \phi \cos^2 \theta \end{bmatrix} \quad (4)$$

In the equation 4,  $I = \text{diag}[I_x, I_y, I_z]^T$  is the representation of the UAV's moment of inertia in the body coordinate system.  $C$  is the Koch force and centrifugal force term, which can be calculated by the following equation:

$$C = J - \frac{1}{2} \frac{\partial}{\partial \theta} (\dot{\theta}^T J) \quad (5)$$

Its specific expression is shown in the literature [3]. In addition,  $d_U$  and  $d_b$  denote the perturbation force and perturbation moment of the airflow on the UAV, respectively.

If the speed of the propellers is  $\omega = [\omega_1, \omega_2, \omega_3, \omega_4]^T$ , then the total lift produced by the four propellers is:

$$U = \sum_{i=1}^4 b \omega_i^2 \quad (6)$$

The rotational moments generated by the four propellers are:

$$\tau = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} 0 & bl & 0 & -bl \\ -bl & 0 & bl & 0 \\ -c & c & -c & c \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (7)$$

where  $b$  and  $c$  are aerodynamic constants and  $l$  is the configuration radius of the UAV. In this paper, the lift force  $U$  and the rotational moment  $\tau$  are used as the control inputs for the corresponding control law design.

Let  $U_p = UR e_3$  be the intermediate control input. Considering the parameter uncertainties,  $m = m_0 + m_\Delta$ ,  $J = J_0 + J_\Delta$ ,  $C = C_0 + C_\Delta$ , the dynamics model of the quadrotor UAV can be rewritten as:

$$\ddot{p} = \frac{U_p}{m_0} - g e_3 + \delta_U \quad (8)$$

$$\ddot{\theta} = J_0^{-1} (\tau - C_0 \dot{\theta}) + \delta_\tau \quad (9)$$

where  $\delta_U = \frac{-m_\Delta \ddot{p} - m_\Delta g e_3 + d_U}{m_0}$  and  $\delta_\tau = J_0^{-1} (-J_\Delta \ddot{\theta} - C_\Delta \dot{\theta} + d_\tau)$  are centralised uncertainties.

### 3. Trajectory Tracking Control Method

Rotary wing UAV trajectory tracking control techniques usually include two main categories: linear control methods and nonlinear control methods.

#### 3.1 Linear Control Methods

In quadrotor UAV trajectory tracking control, common linear control methods include PID control algorithms<sup>[4]</sup>, linear quadratic regulation control algorithms (LQR)<sup>[5]</sup>, and linear quadratic Gaussian regulation control algorithms (LQG)<sup>[6]</sup>. These methods are designed based on a linearised version of the dynamics model, and by linearising the system, the nonlinear problem can be transformed into a linear one, thus simplifying the controller design and analysis process.

Among them, PID controller is a classical linear control method, which consists of three parts, namely, Proportional, Integral and Derivative. PID controller makes the output of the system as close as possible to the desired trajectory by adjusting the parameters of these three parts. Although PID controllers are simple and easy to implement, they require manual parameter adjustments and are less robust to system nonlinearities and external perturbations. LQR is a state-feedback based linear control method, where the state-feedback controllers are designed to minimise the system performance metrics by optimising the linear quadratic cost function. LQR provides optimal control performance, where the system model is known, and the optimum control performance can be calculated by solving the Riccati equation to compute the optimal state feedback matrix. However, LQR requires an accurate system model and is sensitive to measurement noise and modelling errors. The LQG controller is a combination of an LQR controller and a Kalman filter, which achieves the estimation and control of the system state by combining a state estimator with the LQR controller. The LQG controller not only takes into account the state feedback of the system, but also takes into account the state estimation error, and is therefore more robust to systems that contain measurement noise. more robust. However, the LQG controller requires accurate system models and sensor measurements, and has a high computational complexity.

Based on the above analysis, it can be learnt that different linear control methods have their own advantages and disadvantages in quadrotor UAV trajectory tracking control. When selecting linear control methods, we need to consider the dynamic characteristics of the system, performance requirements and the limitations of practical application scenarios.

### 3.2 Non-linear Control Methods

Common nonlinear control methods include model predictive control (MPC)<sup>[7]</sup>, adaptive control<sup>[8]</sup> and robust control<sup>[9]</sup>. These methods provide effective control means for the nonlinear characteristics and complex dynamics models of quadrotor UAV systems.

MPC is a control method for prediction and optimisation based on a mathematical model of the system. For quadrotor UAVs, MPC can use the system's dynamical equations and environmental constraints to predict the system's behaviour over a future period of time and compute the optimal sequence of control inputs through an optimisation algorithm to achieve the desired trajectory tracking. Due to the nonlinear dynamics and multiple degrees of freedom of the quadrotor UAV, the MPC is able to better handle the nonlinear behaviour and dynamic constraints of the system, providing accurate trajectory tracking performance. Adaptive control is a method that can adjust controller parameters in real time according to system changes. For quadrotor UAVs, adaptive control can counter external perturbations and system parameter changes by estimating the system parameters or adjusting the controller gains in real time, thus improving the robustness and stability of the system. This method enables the quadcopter UAV to track the trajectory stably even in the face of wind speed changes and load changes. Robust control is a control method that maintains the stability and performance of the system, even when the system has modelling errors, parameter changes or external disturbances. In quadcopter UAVs, robust control can be achieved by designing stability boundaries, applying robust controllers, or using adaptive techniques to improve the system's resistance to uncertainties and disturbances. This ensures that the quadcopter UAV is able to track a given trajectory stably even in complex environments.

## 4. Numerical Simulation and Hardware Experiments

In the research of rotor UAV trajectory tracking control, MATLAB/Simulink is usually used for numerical simulation to judge whether the controller is effective and feasible by observing whether the tracking errors of position, attitude, etc. converge. In the following, we take the literature<sup>[10]</sup> as an example, and compare the numerical simulation with the classical PD control, and control the UAV to do spiral upward manoeuvre flight. The results are shown in Fig. 2. Literature<sup>[10]</sup> uses BP neural network PID control, compared with the classical PID control, BP neural network PID control step response is rapid, and does not produce overshoot and oscillation. The BP neural network PID control

in the improved algorithm can avoid overshooting and oscillation in the process of rapid response and make the system more stable. It can be observed from Fig. 2 that the blue curve has higher convergence accuracy and faster convergence speed than the red curve, and the curve is more stable in the fluctuation in the neighbourhood of zero, which verifies the superiority of the BPPID control method.

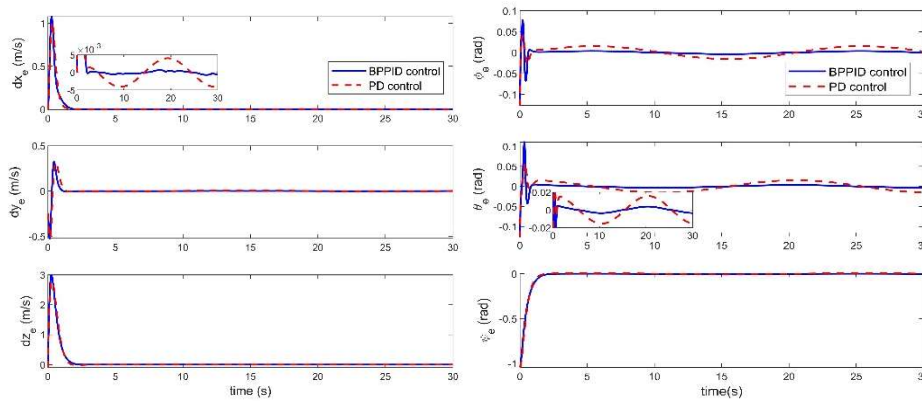


Fig. 2 Tracking error of some parameters of the UAV control system

The hardware experimental platform usually includes a UAV, an on-board computer, a remote control, and a ground control station, as shown in Fig. 3.

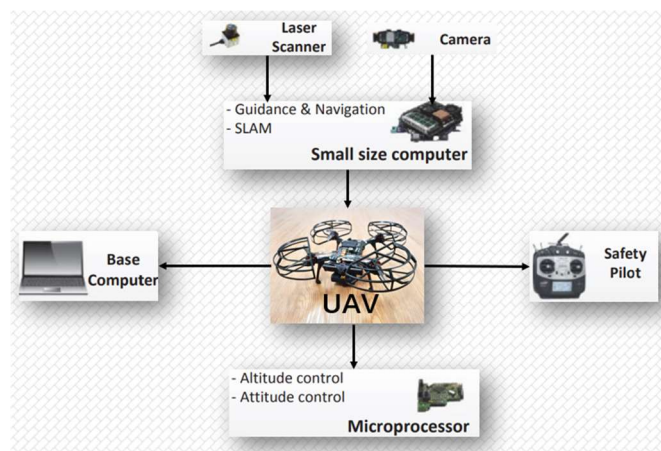


Fig. 3 Hardware platform for UAV flight experiment

## 5. Conclusion

This paper reviews and analyses the research on trajectory tracking control of rotary-wing UAVs, focusing on four aspects, including UAV modelling, control strategy, numerical simulation and hardware experimental flight platform. The study shows that in the trajectory tracking control of rotary-wing UAVs, scholars currently use a variety of control strategies, including linear and nonlinear control methods, etc., and different control strategies have their own advantages and limitations in specific flight missions and environments. However, rotary wing UAV trajectory tracking control also faces many challenges in terms of nonlinear dynamics characteristics, external environmental perturbations, fast dynamic response, path planning and control integration, real-time and computational complexity, safety and reliability. Solving these challenges will require comprehensive consideration of knowledge from multiple fields such as control theory, flight dynamics, path planning algorithms, real-time system technology, etc., and is a worthy topic for in-depth research in the future.

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