Research on Interference between a Piezoelectric Screw Dislocation Dipole and Nano-inclusions

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Abstract

The electro-elastic coupling effect of dislocation dipoles in piezoelectric matrix and the circular nanoscle inclusion with interface effect was investigated. Using the complex potential method of Elasticity, the analytical solution of matrix and nanoscale inhomogeneity is obtained firstly, then the stress field of matrix and inclusion, the electric displacement field, and the image force acting on the screw dipole center are deduced. Numerical calculation shows that the existence of interface effect can attenuate attractive force of soft inclusion on dislocation dipole and enhance the rejection of hard inclusion on dislocation dipole, illustrating that the interface effect produces rejection force on screw dislocation dipole.

Keywords

Screw Dislocation Dipole; Nanoscale Inclusion; Interface Effect; Piezoelectric Materials; Complex Variable Method.

1. Introduction

Piezoelectric materials generate electric fields due to mechanical deformation under the action of force, and can also generate mechanical deformation under pure electric field. This electromechanical coupling effect makes piezoelectric composite materials widely used in engineering, especially in the field of intelligent structures that require self bearing capacity, self diagnosis, self adaptability, and self repair function. However, during the preparation and use of piezoelectric composite materials, it is difficult to avoid micro defects such as inclusions, rigid nuclei, dislocations, cracks, and pores. These defects not only appear in the matrix and inclusions, but also on the interface and surface. Their existence inevitably affects the various properties of piezoelectric composite materials during service. Therefore, establishing a reasonable mechanical model to study the mutual interference mechanism between micro defects in piezoelectric composite materials from a microscopic perspective is of great theoretical significance and practical value[1-9].

Dislocations and dislocation dipoles are common microscopic defects in crystalline materials. Dislocation dipoles are composed of two equally sized (Burgers vectors) dislocations with opposite directions. The stress field generated by them is much smaller than that generated by a single dislocation, making them more likely to occur in materials, and their impact on the material cannot be ignored [10, 11]. When studying the interaction mechanism between dislocation dipoles and inclusions, the size of inclusions and the boundary conditions of interfaces are crucial. For larger inclusions (micrometer scale and above), the interface occupied area is very small compared to the size of the inclusion area, and the interface effect can be ignored. However, for very small inclusions (such as nanometer scale), the interface occupied area cannot be ignored compared to the inclusion area. In this case, the interface stress effect must be considered [12-16]. The "surface/interface effect model" proposed by Gurtin and Murdoch [15], also known as the interface stress model, is widely used in the study of nanomaterials or nanoscale structural mechanics. This model considers the interface as a region bonded to an inclusion, with different elastic moduli and constitutive equations from the inclusion, and does not consider thickness. The influence of interface effects on the matrix and inclusions is achieved by constructing local perturbation boundary conditions based on interface stress. Fang [12] studied the elastic interference between screw dislocations and nanocircular inclusions containing interfacial stress. Pan [16] investigated the variation of effective elastic modulus in piezoelectric materials under interface effects; Xu [10] investigated the electroelastic coupling interference effect between a single screw dislocation and a circular nano inclusion containing interface effects. The previous research mainly focused on studying individual dislocations, and there have been no reports on the interference between dislocation dipoles and nano-inclusions. This article establishes a mechanical model for the interference between spiral dislocation dipoles and circular nano piezoelectric inclusions containing interface effects. Using the complex potential method of elastic mechanics, the stress fields of the matrix, inclusions, and interfaces, as well as the series form analytical solutions of the dislocation image force acting at the center of the dislocation dipole, are derived. The influence of interface effects, material parameters, and other factors on the dislocation image force is discussed through numerical analysis.

2. Basic Formulas and Problem Description

2.1 Basic Formulas

For a transversely isotropic piezoelectric medium with polarization direction along the axis, its isotropic plane is set as plane xoy. Under the action of anti-plane loading and in-plane electric field at infinity, only anti plane displacement w, stress components τ_{xz} and τ_{yz}, strain components γ_{xz} and γ_{yz}, as well as electric potential ϕ, electric displacement components D_x and D_y, and electric field strength E_x and E_y are generated. Let the generalized displacement vector is $\mathbf{U}_j=\left[w_j,\varphi_j\right]^T$, generalized stress vectors are $\mathbf{\Sigma}_{xzj}=\left[\tau_{xzj},D_{xj}\right]^T$ and $\mathbf{\Sigma}_{yzj}=\left[\tau_{yxzj},D_{yj}\right]^T$, and generalized strain vectors are $\mathbf{Y}_{xzj}=\left[\varepsilon_{xzj},E_{xj}\right]^T$ and $\mathbf{Y}_{yzj}=\left[\varepsilon_{yzj},E_{yj}\right]^T$, according to the complex potential method in Elasticity, these physical quantities can be represented by the complex potential function $\mathbf{f}_j(z)=\left[f_{wj}(z),f_{\phi j}(z)\right]^T$ as follows [9-11]:

$$\mathbf{U}=\mathrm{Re}\,\mathbf{f}(z) \tag{1}$$

$$\mathbf{\Sigma}_{xz}-\mathrm{i}\mathbf{\Sigma}_{yz}=\mathbf{MF}(z) \tag{2}$$

$$\mathbf{Y}_{xz}-\mathrm{i}\mathbf{Y}_{yz}=\mathbf{F}(z) \tag{3}$$

where $\mathbf{F}(z)=\mathbf{f}'(z)$, $\mathbf{M}$ is the electroelastic modulus matrix, and $\mathbf{M}=\begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -d_{11} \end{bmatrix}$.

When taking polar coordinates, Eq. (2) can be written as:

$$\mathbf{\Sigma}_{\rho}-\mathrm{i}\mathbf{\Sigma}_{\theta}=\mathrm{e}^{\mathrm{i}\theta}\mathbf{MF}(z) \tag{4}$$

2.2 Problem Description

As shown in Fig. 1, there is a circular nano inclusion (region S^+) with a radius of R in the infinite piezoelectric medium matrix (region S^-). The interface between the inclusion and the matrix is marked with L, and the matrix is subjected to an anti plane force field ($\tau_{xz}^{\infty}, \tau_{yz}^{\infty}$) and an in-plane electric field ($D_x^{\infty}, D_y^{\infty}$) at infinity. Let the matrix and inclusions be transversely isotropic, with the xoy plane as the isotropic plane, and the inclusions extend infinitely in the z direction. The center of the piezoelectric screw dislocation dipole is located at point $z_0(=x_0+iy_0=\rho e^{i\theta})$ in the matrix, containing two generalized screw dislocations $b_1=b=\left[b_z, b_\varphi\right]^T$ and $b_2=-b=\left[-b_z,-b_\varphi\right]^T$ located at points $z_1=z_0-de^{i\varphi}$ and $z_2=z_0+de^{i\varphi}$ respectively (where is the dipole arm length $2d$, and the inclination angle φ is the angle between the dipole arm and the positive half axis of the x-axis).

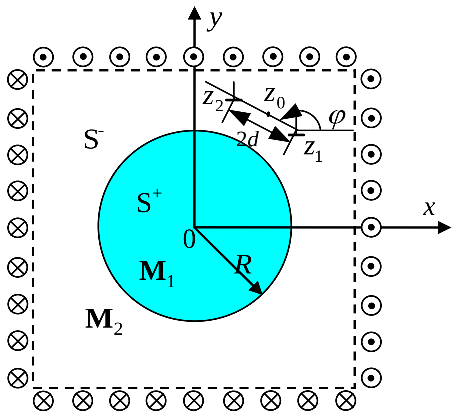

Fig.1 Interference model between a piezoelectric screw dislocation dipole and a circular inclusion with interface effect

Assuming the center of the circular inclusion is the origin $z=x+iy$ on the complex plane, representing point $t=Re^{i\theta}$ on the interface L ($|z|=R$). The interface connection conditions for this problem can be represented by the generalized displacement vector F and the generalized stress as [9, 13]:

$$\mathbf{U}_1^+(t)=\mathbf{U}_2^-(t) \qquad t\in L \tag{5}$$

$$\mathbf{\Sigma}_{r1}^+(t)-\mathbf{\Sigma}_{r2}^-(t)=\frac{-1}{R}\mathbf{M}^s\frac{\partial}{\partial\theta}\mathrm{Im}\left[e^{i\theta}f'(z)\right] \ t\in L \tag{6}$$

where $\mathbf{M}^s=\begin{bmatrix} c_{44}^s-\tau_0^s & e_{15}^s \\ e_{15}^s & -d_{11}^s \end{bmatrix}$ is the matrix of the electrical elastic modulus for the interface, τ_0^s is the residual stress, c_{44}, e_{15}, d_{11} represent the longitudinal shear modulus (under constant electric field), piezoelectric constant and dielectric constant (under constant stress field) of piezoelectric materials, respectively, R is the radius of nano-inclusions. The subscripts "1" and "2" represent the region (inclusion) and (matrix), respectively, while the superscripts "+" and "−" represent the values taken by the function as it approaches the interface from S^+ and S^-.

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3. Solving of the Problem

When the screw dislocation dipole is located in the matrix, singularity analysis shows that the complex potential function of the region can be expressed as:

$$\mathbf{F}_2(z)=\mathbf{B}\left[\frac{1}{z-z_1}-\frac{1}{z-z_2}\right]+\mathbf{\Gamma}+\mathbf{F}_{20}(z),\quad z\in \mathrm{S}^- \tag{7}$$

where $\mathbf{B}=\frac{1}{2\pi \mathrm{i}}\mathbf{b}=\frac{1}{2\pi \mathrm{i}}\left[b_z,b_\varphi\right]^{\mathrm{T}}$ is the dislocation strength for generalized spiral dislocations. $\mathbf{F}_{20}(z)$ is a holomorphic function in the region, while Γ is determined by the stress and potential shift at infinity, which can be written as:

$$\mathbf{\Gamma}=\mathbf{M}_2^{-1}(\mathbf{\Sigma}_x^\infty-i\mathbf{\Sigma}_y^\infty)=\mathbf{M}_2^{-1}\begin{bmatrix}\tau_{xz}^\infty-\mathrm{i}\,\tau_{yz}^\infty\\ D_x^\infty-\mathrm{i}D_y^\infty\end{bmatrix} \tag{8}$$

where $\mathbf{M}_2=\begin{bmatrix}\mathrm{c}_{44}^{(2)} & \mathrm{e}_{15}^{(2)}\\ \mathrm{e}_{15}^{(2)} & -\mathrm{d}_{11}^{(2)}\end{bmatrix}$, and the superscrip “$-1$” indicates matrix inversion.

Based on the extended Schwartz analytic extension principle, two new analytical functions $\mathbf{F}_{1*}(z)$ and $\mathbf{F}_{2*}(z)$ are defined, and it is noted that there is $t\bar{t}=R^2$ on $|z|=R$, where “—” represents conjugation of complex numbers.

$$\mathbf{F}_{1*}(z)=-\frac{R^2}{z^2}\overline{\mathbf{F}}_1\left(\frac{R^2}{z}\right)\qquad z\in \mathrm{S}^- \tag{9}$$

$$\mathbf{F}_{2*}(z)=-\frac{R^2}{z^2}\overline{\mathbf{F}}_2\left(\frac{R^2}{z}\right)\qquad z\in \mathrm{S}^+ \tag{10}$$

Substituting Eq. (7) into Eq. (10) yields:

$$\mathbf{F}_{2*}(z)=-\mathbf{B}\left(\frac{1}{z-z_1^*}-\frac{1}{z-z_2^*}\right)-\frac{R^2}{z^2}\overline{\mathbf{\Gamma}}+\mathbf{F}_{20*}(z)\qquad z\in \mathrm{S}^+ \tag{11}$$

In the formula, $z_i^*=R^2/\overline{z_i}\,(i=1,2)$ is the holomorphic function within the region S^+.

Substitute Eq.(1) for differentiation into Eq.(5), then:

$$\left[\mathbf{F}_1(t)-\mathbf{F}_{2*}(t)\right]^+=[\mathbf{F}_2(t)-\mathbf{F}_{1*}(t)]^-\qquad t\in \mathrm{L} \tag{12}$$

Consider Eqs. (8)-(12), and from the generalized Liouville theorem, it can be obtained that:

$$\mathbf{F}_1(z)-\mathbf{F}_{2*}(z)=g(z)\qquad z\in \mathrm{S}^+ \tag{13}$$

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$$\mathbf{F}_2(z) - \mathbf{F}_{1*}(z) = g(z) \qquad z \in \mathrm{S}^- \tag{14}$$

On the entire plane, there is:

$$g(z) = \mathbf{B}\left[\frac{1}{z-z_1} - \frac{1}{z-z_2} + \frac{1}{z-z_1^*} - \frac{1}{z-z_2^*}\right] + \mathbf{\Gamma} + \frac{R^2}{z^2}\overline{\mathbf{\Gamma}} \tag{15}$$

Substitute Eq.(2) into Eq. (6), and consider the Eqs.(7)-(11) yields:

$$[\mathbf{M}_1\mathbf{F}_1(t) - \mathbf{M}_2\mathbf{F}_{2*}(t) + \frac{\mathbf{M}^{\mathrm{s}}}{R}\mathbf{F}_{2*}(t)]^+ = [\mathbf{M}_2\mathbf{F}_2(t) - \frac{\mathbf{M}^{\mathrm{s}}}{R}\mathbf{F}_2(t) + \mathbf{M}_1\mathbf{F}_{1*}(t) - \frac{1}{R}\mathbf{M}^{\mathrm{s}}t\mathbf{F}_2{}'(t) - \frac{1}{R}\mathbf{M}^{\mathrm{s}}\mathrm{e}^{-2\mathrm{i}\theta}\overline{t\mathbf{F}'}\left(\frac{R^2}{t}\right)]^- \tag{16}$$

Differentiating Eq.(10) over z yields:

$$\overline{\mathbf{F}_2{}'(z)} = \mathrm{e}^{4\mathrm{i}\theta}\mathbf{F}_{2*}{}'(z) - \frac{2\mathrm{e}^{\mathrm{i}\theta}}{R}\overline{\mathbf{F}(z)} \tag{17}$$

Substituting Eq.(17) into equation Eq.(16) yields:

$$\left[\mathbf{M}_1\mathbf{F}_1(t) + \left(\mathbf{M}_2 + \frac{\mathbf{M}^{\mathrm{s}}}{R}\right)\mathbf{F}_{2*}(t) + \frac{t\mathbf{M}^{\mathrm{s}}}{R}\mathbf{F}_{2*}{}'(t)\right]^+ = \left[\mathbf{M}_1\mathbf{F}_{1*}(t) + \left(\mathbf{M}_2 - \frac{1}{R}\mathbf{M}^{\mathrm{s}}\right)\mathbf{F}_2(t) - \frac{t\mathbf{M}^{\mathrm{s}}}{R}\mathbf{F}_2{}'(t)\right]^- \qquad t \in \mathrm{L} \tag{18}$$

From the generalized Liouville theorem and Eq.(18), it can be obtained that:

$$\mathbf{M}_1\mathbf{F}_1(z) + \left(\mathbf{M}_2 + \frac{\mathbf{M}^{\mathrm{s}}}{R}\right)\mathbf{F}_{2*}(z) + \frac{t\mathbf{M}^{\mathrm{s}}}{R}\mathbf{F}_{2*}{}'(z) = \mathbf{h}(z) \qquad z \in \mathrm{S}^+ \tag{19}$$

$$\mathbf{M}_1\mathbf{F}_{1*}(z) + \left(\mathbf{M}_2 - \frac{\mathbf{M}^{\mathrm{s}}}{R}\right)\mathbf{F}_2(z) - \frac{z\mathbf{M}^{\mathrm{s}}}{R}\mathbf{F}_2{}'(z) = \mathbf{h}(z) \qquad z \in \mathrm{S}^- \tag{20}$$

And from Eqs.(8)-(11), it can be obtained that on the entire plane:

$$\begin{aligned}\mathbf{h}(z) = {} & \left(\mathbf{M}_2 - \frac{\mathbf{M}^{\mathrm{s}}}{R}\right)\mathbf{B}\left(\frac{1}{z-z_1} - \frac{1}{z-z_2}\right) + \left(\mathbf{M}_2 - \frac{\mathbf{M}^{\mathrm{s}}}{R}\right)\mathbf{\Gamma} + \frac{\mathbf{M}^{\mathrm{s}}}{R}\mathbf{B}\left[\frac{z}{(z-z_1)^2} - \frac{z}{(z-z_2)^2}\right] + \frac{2R}{z^2}\mathbf{M}^{\mathrm{s}}\overline{\mathbf{\Gamma}} \\ & - \left(\mathbf{M}_2 + \frac{\mathbf{M}^{\mathrm{s}}}{R}\right)\mathbf{B}\left(\frac{1}{z-z_1^*} - \frac{1}{z-z_2^*}\right) - \left(\mathbf{M}_2 + \frac{\mathbf{M}^{\mathrm{s}}}{R}\right)\frac{R^2}{z^2}\overline{\mathbf{\Gamma}} + \frac{\mathbf{M}^{\mathrm{s}}}{R}\mathbf{B}\left[\frac{z}{(z-z_1^*)^2} - \frac{z}{(z-z_2^*)^2}\right]\end{aligned} \tag{21}$$

Combining Eq.(11) and Eq.(19) yields:

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$$[\mathbf{M}_1+\mathbf{M}_2+\frac{\mathbf{M}^s}{R}]F_1(z)+\frac{\mathbf{M}^s}{R}zF_1'(z)=\mathbf{h}(z)+\left[\mathbf{M}_2+\frac{\mathbf{M}^s}{R}\right]\mathbf{g}(z)+\frac{\mathbf{M}^s}{R}z\mathbf{g}'(z) \tag{22}$$

In region S^+, expand $\mathbf{F}_1(z)$ into Taylor series as:

$$\mathbf{F}_1(z)=\sum_{k=0}^{\infty}a_k z^k \qquad |z|<R \tag{23}$$

According to Eq. (22), the constants can be obtained as:

$$a_0=-2\left[\mathbf{M}_1+\mathbf{M}_2+\frac{\mathbf{M}^s}{R}\right]^{-1}\mathbf{M}_2\left[\mathbf{B}\left(\frac{1}{z_1}-\frac{1}{z_2}\right)-\mathbf{\Gamma}\right] \tag{24}$$

$$a_k=-2\left[\mathbf{M}_1+\mathbf{M}_2+\frac{1+k}{R}\mathbf{M}^s\right]^{-1}\mathbf{M}_2\mathbf{B}\left(\frac{1}{z_1^{k+1}}-\frac{1}{z_2^{k+1}}\right) \quad k\geq 1 \tag{25}$$

Substituting Eq. (23) into Eq. (10) has:

$$\mathbf{F}_{1*}(z)=-\sum_{k=0}^{\infty}\overline{a_k}R^{2(k+1)}z^{-k-2} \quad |z|>R \tag{26}$$

Substituting Eq. (26) into Eq. (14) has:

$$\begin{aligned}\mathbf{F}_2(z)=&\mathbf{B}\left[\frac{1}{z-z_1}-\frac{1}{z-z_2}+\frac{1}{z-z_1^*}-\frac{1}{z-z_2^*}\right]+\mathbf{\Gamma}+\frac{R^2}{z^2}\overline{\mathbf{\Gamma}}\\&+2\left[\mathbf{M}_1+\mathbf{M}_2+\frac{\mathbf{M}^s}{R}\right]^{-1}\mathbf{M}_2\left[-\mathbf{B}\left(\frac{1}{\overline{z}_1}-\frac{1}{\overline{z}_2}\right)-\overline{\mathbf{\Gamma}}\right]\frac{R^2}{z^2} \qquad |z|>R\\&-\sum_{k=0}^{\infty}2\left[\mathbf{M}_1+\mathbf{M}_2+\frac{1+k}{R}\mathbf{M}^s\right]^{-1}\mathbf{M}_2\mathbf{B}\left(\frac{1}{\overline{z}_1^{k+1}}-\frac{1}{\overline{z}_2^{k+1}}\right)R^{2k+2}z^{-k-2}\end{aligned} \tag{27}$$

The generalized stress field expression inside the nano inclusion can be obtained from Eq. (2) and Eq. (23), which is:

$$\begin{aligned}\mathbf{\Sigma}_{xz1}-\mathrm{i}\mathbf{\Sigma}_{yz1}=&-2\mathbf{M}_1\left[\mathbf{M}_1+\mathbf{M}_2+\frac{\mathbf{M}^s}{R}\right]^{-1}\mathbf{M}_2\left[\mathbf{B}\left(\frac{1}{z_1}-\frac{1}{z_2}\right)-\mathbf{\Gamma}\right]\\&-\sum_{k=0}^{\infty}2\mathbf{M}_1\left[\mathbf{M}_1+\mathbf{M}_2+\frac{1+k}{R}\mathbf{M}^s\right]^{-1}\mathbf{M}_2\mathbf{B}\left(\frac{1}{z_1^{k+1}}-\frac{1}{z_2^{k+1}}\right)z^k\end{aligned} \qquad z\in S^+ \tag{28}$$

The generalized stress field expression within the matrix can be obtained from Eq. (2) and Eq. (27), which is:

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$$\begin{aligned}\mathbf{\Sigma}_{xz2}-\mathrm{i}\mathbf{\Sigma}_{yz2}=&\mathbf{M}_2\mathbf{B}\left[\frac{1}{z-z_1}-\frac{1}{z-z_2}+\frac{1}{z-z_1^*}-\frac{1}{z-z_2^*}\right]+\mathbf{M}_2\mathbf{\Gamma}+\frac{R^2}{z^2}\mathbf{M}_2\bar{\mathbf{\Gamma}}\\&+2\mathbf{M}_2\left[\mathbf{M}_1+\mathbf{M}_2+\frac{\mathbf{M}^{\mathrm{s}}}{R}\right]^{-1}\mathbf{M}_2\left[-\mathbf{B}\left(\frac{1}{\bar{z}_1}-\frac{1}{\bar{z}_2}\right)-\bar{\mathbf{\Gamma}}\right]\frac{R^2}{z^2}\qquad z\in\mathrm{S}^-\\&-\sum_{k=0}^{\infty}2\mathbf{M}_2\left[\mathbf{M}_1+\mathbf{M}_2+\frac{1+\mathrm{k}}{R}\mathbf{M}^{\mathrm{s}}\right]^{-1}\mathbf{M}_2\mathbf{B}\left(\frac{1}{\bar{z}_1^{k+1}}-\frac{1}{\bar{z}_2^{k+1}}\right)R^{2k+2}z^{-k-2}\end{aligned}\tag{29}$$

When there are no dislocations, substituting Eq.(27) into Eq.(4) can obtain the stress field and potential shift field at the interface as:

$$\mathbf{\Sigma}_{\rho}-\mathrm{i}\mathbf{\Sigma}_{\theta}=\mathrm{e}^{\mathrm{i}\theta}\mathbf{M}_2\mathbf{F}_2(t)=\mathrm{e}^{\mathrm{i}\theta}\mathbf{M}_2\mathbf{\Gamma}+\left[\mathbf{M}_1+\mathbf{M}_2+\frac{\mathbf{M}^{\mathrm{s}}}{R}\right]^{-1}-2\mathrm{e}^{\mathrm{i}\theta}\mathbf{M}_2\left[\mathbf{M}_1+\mathbf{M}_2+\frac{\mathbf{M}^{\mathrm{s}}}{R}\right]^{-1}\mathbf{M}_2\bar{\mathbf{\Gamma}}\frac{R^2}{t^2}\tag{30}$$

The sum of the image force components acting on the center of the screw dislocation dipole can be given by the following equation [17]:

$$F_x-iF_y=F_{x1}+F_{x2}-i(F_{y1}+F_{y2})\tag{31}$$

According to the Peach-Koehler formula, the calculation formula for F_{xj} , F_{yj} ($j=1,2$) is $F_{xj}-iF_{yj}=i\mathbf{b}_j^T\left(\tilde{\mathbf{\Sigma}}_{x1}^j-i\tilde{\mathbf{\Sigma}}_{y1}^j\right)$ $j=1,2$. Among them, $\tilde{\mathbf{\Sigma}}_{x1}^j$ and $\tilde{\mathbf{\Sigma}}_{y1}^j$ represent the stress field of screw dislocation at point z_j minus the stress field generated by the corresponding screw dislocation in an infinitely uniform matrix. And take the limit value of the stress field for $z\to z_j$ to obtain the perturbation stress field of the dislocation point. The analytical expressions for dislocation image force and image force couple moment can be obtained from Eqs. (27)-(31).

Without loss of generality, the following assumptions are made during the calculation of the example: loading at infinity is zero, namely $\mathbf{\Gamma}=0$; the center of the dislocation dipole is located at a certain point on the x-axis ($z_0=x_0>R$), and only the dislocation component force F_x along the x direction of the dislocation dipole center in the matrix is discussed, which is dimensionless as $F_{x0}=2\pi R\left[\mathbf{b}_T\mathbf{M}_2\mathbf{b}\right]^{-1}F_x$.

4. Numerical Analysis

Assuming the matrix material is piezoelectric ceramic PZT-5H, its stiffness modulus matrix is $\mathbf{M}_2=\begin{bmatrix}12.6\times10^{10}\,\mathrm{N/m^2} & 6.5\mathrm{C/m^2}\\ 6.5\mathrm{C/m^2} & -1.51\times10^{-8}\mathrm{C/Vm}\end{bmatrix}$, the stiffness modulus matrix of the interface is $\mathbf{M}^{\mathrm{s}}=\begin{bmatrix}7.56\mathrm{N/m} & 3\times10^{-8}\mathrm{C/m}\\ 3\times10^{-8}\mathrm{C/m} & 0\end{bmatrix}$, and the piezoelectric screw dislocation is $\mathbf{b}=\left[1.0\times10^{-9}\mathrm{m}\ 1.0\mathrm{V}\right]^{\mathrm{T}}$. The dielectric constant of nano piezoelectric inclusions and piezoelectric matrix is set as $\mathrm{d}_{11}^{(1)}=\mathrm{d}_{11}^{(2)}$. The following dimensionless quantities are defined as $u=\mathrm{c}_{44}^{(1)}/\mathrm{c}_{44}^{(2)}$, $v=\mathrm{e}_{15}^{(1)}/\mathrm{e}_{15}^{(2)}$, and $\mathrm{e}_{15}^{\mathrm{s}}=\delta\mathrm{e}_0^{\mathrm{s}}$, where $\mathrm{e}_0^{\mathrm{s}}=3\times10^{-8}\mathrm{C/m}$.

Fig.2 shows the variation of F_{x0} with the relative position z_0/R of the center of the dislocation dipole when $\varphi=\pi/2$, $v=1$, $\varphi=\pi/2$, $R=10\mathrm{nm}$, $d=5R$. As shown in the figure, when the interface effect does not exist ($\mathbf{M}_s=0$), the nano soft inclusion ($u<1$) always attracts dislocation dipoles in the matrix,

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while the nano hard inclusion ($u>1$) consistently repels dislocation dipoles in the matrix, consistent with the conclusions of classical elastic theory. When there is an interface effect ($\mathbf{M}_s \neq 0$), it weakens the attraction of soft inclusions to dislocations and enhances the repulsive force of hard inclusions to dislocations, indicating that the interface effect has a repulsive effect on dislocations.

Fig.3 shows the variation of F_{x0} with respect to the relative position d/R of the dislocation dipole center when $R=10\text{nm}$, $v=1$, $\varphi=\pi/6$, $z_0=8R$. As shown in the figure, when the nano-inclusions are relatively hard, F_{x0} remains positive, indicating that the dislocation dipole is repelled by the hard inclusions, and the presence or absence of interface effects has little effect on it. When the nano-inclusions are relatively soft, F_{x0} remains negative, indicating that the dislocation dipole is attracted by the soft inclusions. When the interface effect exists, the attraction force F_{x0} decreases, indicating that the interface effect can weaken the attraction of the soft inclusions to dislocations, which is consistent with the analysis results in the above figure.

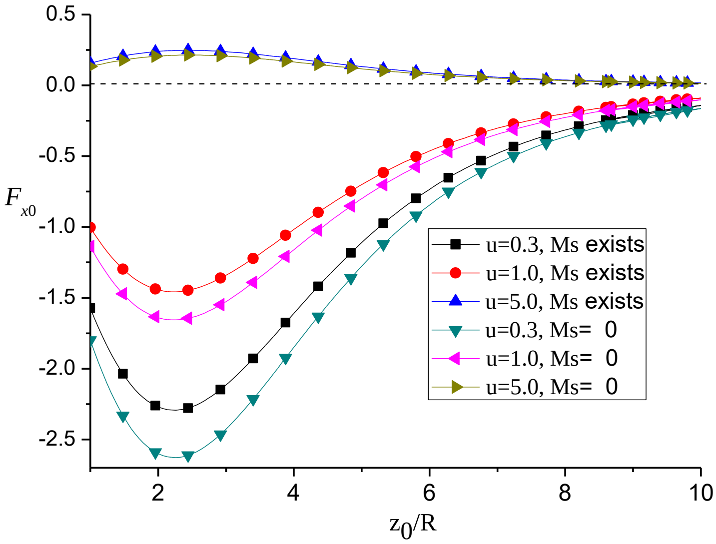

Fig. 2 The variation of F_{x0} with the relative position of dipole center z_0/R

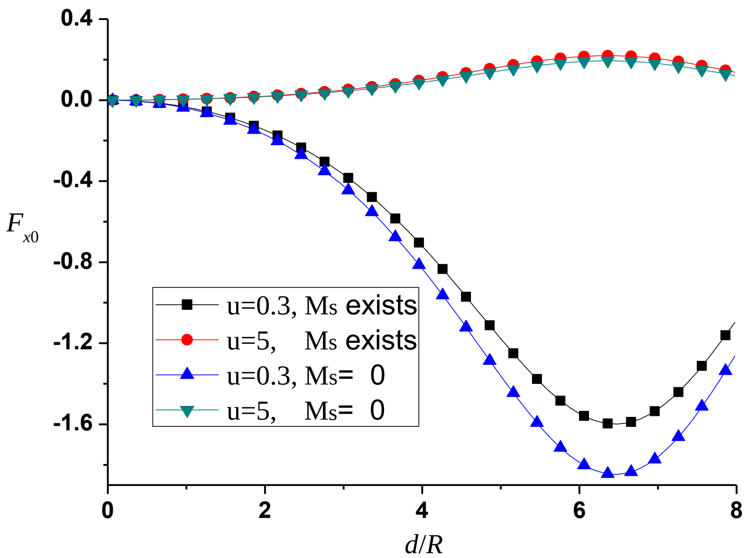

Fig. 3 The variation of F_{x0} with the relative position of dipole center d/R

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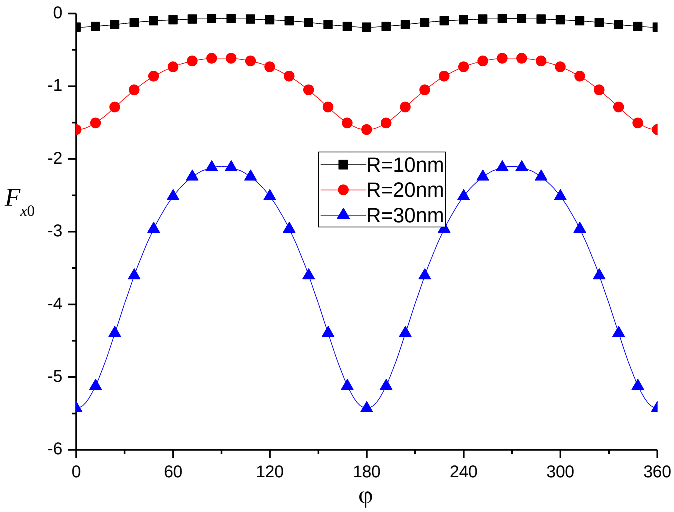

Fig. 4 The variation of F_{x0} with dipole angle φ of aislocation aipoles

Fig. 4 shows the variation of F_{x0} with the dip angle φ of the dislocation dipole for different radius R when $u=1$, $v=1$, $d=40\,\mathrm{nm}$, $z_0=100\,\mathrm{nm}$. As shown in the figure, the dislocation image force F_{x0} acting at the center of the dislocation dipole varies periodically with the dipole inclination angle φ. When $\varphi\in(0,\pi/2)$, as φ increases, the attraction force F_{x0} decreases; when $\varphi\in(\pi/2,\pi)$, as φ increased, the attraction force F_{x0} increased. When φ is constant, the the attraction force F_{x0} increases with the increase of nano-inclusions.

5. Summary

This paper investigates the electromechanical coupling effect between a screw dislocation dipole and a nanoscale inclusions containing interface stress in piezoelectric materials. Using the complex potential function method of elasticity and dislocation theory, the stress fields of the matrix and region, as well as the analytical expressions for the image force and image force moment acting on the center of the dislocation dipole, were obtained. The influence of relative shear modulus, size of nano-inclusions, dipole inclination angle and position on the dislocation image force was analyzed using examples. The results show that when the nano-inclusions are relatively hard, the dislocation image force remains positive, indicating that the dislocation dipole is repelled by the hard inclusions, and the interface effect has little effect on it; When the nano-inclusions are relatively soft, the dislocation image force remains negative; When the interface effect exists, it weakens the attraction of soft inclusions to dislocations and enhances the repulsive force of hard inclusions to dislocations, indicating that the interface effect has a repulsive effect on dislocations.

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